MAS160: Signals, Systems & Information for Media Technology

Problem Set 3

Problem 1: Basis Functions

It is possible to use the definition of orthonormality to derive sets of basis functions. What we will use here is a simple version of what is known as the Graham-Schmidt Procedure.

Assume we care about our functions only on the interval $(-1 \leq t \leq 1)$. First we choose some basis function $\phi_0(t)$ such that it satisfies the requirement

$$\int_{-1}^{1} \phi_0^2(t)dt = 1.$$

Then a second member of the basis set, $\phi_1(t)$, must satisfy

$$\int_{-1}^{1} \phi_1^2(t)dt = 1$$

and

$$\int_{-1}^{1} \phi_0(t)\phi_1(t)dt = 0;$$

a third member $\phi_2(t)$ must satisfy three equations, and so forth.

Consider (on the interval from -1 to 1) the basis functions

$$\phi_0(t) = A \ ,$$

$$\phi_1(t) = Bt + C \text{ and}$$

$$\phi_2(t) = Dt^2 + Et + F \ .$$

(a) What are the values of A, B, C, D, E and F needed to make these orthonormal on this interval?

(b) Sketch these three basis functions in the interval $(-1 \leq t \leq 1)$.

(c) What are the coefficients for a series approximation (using $\phi_0(t)$, $\phi_1(t)$, and $\phi_2(t)$) of the function $p(t) = 1 + \cos(\pi t)$ for $-1 \leq t \leq 1$?

These functions, the $\phi_i(t)$'s, are known as Legendre polynomials, and a tremendous amount is known about them. If you're interested you can find out much, much more at mathworld. http://mathworld.wolfram.com/LegendrePolynomial.html

Problem 2: AM and Sampling

(*DSP First* 4.6)

Problem 3: Frequency, Sampling and Bit Rate

The high-frequency limit of human hearing extends to approximately 20,000 Hz, but studies have shown that intelligible speech requires frequencies only up to 4,000 Hz.

(a) Justify why the sampling rate for an audio Compact Disc (CD) is 44.1 kHz.

(b) What is the Nyquist rate for reliable speech communications? Why do you think people sound different on the phone from in person?

The bit rate of a system can be calculated quite simply as follows:

$$\text{bit rate} = (\text{sampling rate})\ (\text{number of bits per sample})$$

(c) Suppose intelligible speech requires 7 bits per sample. If the phone system is designed to just meet the requirements for speech (which is the case), what is the maximum bit rate allowable over telephone lines? From your result, do you think computer modems (not cable modems, ISDN, or DSL) will get any faster?

(d) CDs use 16 bits per sample. What is the bit rate of music coming off a CD? Is a modem connection fast enough to support streamed CD quality audio?

Problem 4: Non-ideal D-to-C Conversion

(*DSP First* 4.8)

Problem 5: Representing Irrational Frequencies (for MAS 510)

Later in this course we will describe how if a signal is periodic in time, then it is discrete in frequency, and vice-versa. At first glance, Fig 3.17 seems to violate this statement; however, if you look closer, it does not. Let's explore what is going on.

Consider the following signals

$$x(t) = 2\cos(10\pi\sqrt{8}t) + 3\cos(30\pi\sqrt{27}t) \quad (1)$$

$$y(t) = 2\cos(10\pi t) + 3\cos(30\pi t) \quad (2)$$

(a) Plot the two signals in the time domain on the same page.

(b) Plot the two signals in the frequency domain using the `stem` function in MATLAB.

Is x(t) really discrete in the frequency domain? What is the computer's approximation of the irrational frequencies?