Problem 1: Return of the rabbits!

Consider the case of the Fibonacci rabbits: They become fertile at one month of age, they have a gestation period of one month, each litter is one male and one female, and they live and breed "forever," from our perspective.

From that description, we could write the following difference equation for the output of a system:

$$r[n] = r[n-1] + r[n-2],$$

where r is pairs of rabbits and n is months, and we will introduce 1 pair of rabbits into the system at $n = 1$ (more explicitly, for $n \leq 0$, $r[n] = 0$, and $r[1] = 1$).

(a) Let's think about this a bit more explicitly in z-transform terms. Where are the poles and zeroes of this system?

(b) Suppose we add a new twist: only P percent of the fertile (*i.e.* at-least-two-month-old) rabbits give birth in any given month. Rewrite the difference equation. Now where are the poles and zeroes? *If it makes your life easier, from this point on you can forget that r must always be an integer. If you are looking for a challenge, try to retain that constraint!*

Problem 2: Inverse z-Transforms

Determine all possible signals that can have the following z-transforms with the given conditions.

(a) $\frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$

(b) $\frac{2 - \frac{3}{2}z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$, causal

(c) $\frac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}}$, stable

(d) $\frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{1}{2}z^{-1}}$, right-handed

Problem 3: Utilizing the z-transform (*DSP First* 8.12)

Problem 4: MAS 510 Additional Problem

(All-Pass System) Consider the causal linear shift-invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}}$$

where a is real

(a) For what range of values of a is the system stable?

(b) If $0 < a < 1$, plot the pole-zero diagram and shade the region of convergence. Do the same for $a > 1$.

(c) $H(z)$ is to be cascaded with a system $\hat{H}(z)$ so that the overall system function is unity. With $0 < a < 1$ and $\hat{H}(z)$ specified to be a stable system, determine its impulse response $\hat{h}(n)$.

Problem 5: Discrete Fourier Transforms (*DSP First* **9.2**)

Problem 6: Inverse DFT (*DSP First* **9.3**)

Problem 7: Convolution revisited

(a) In MATLAB, create two vectors as follows:

```
x1 = [1 1 1 1 0 0 0 0];
x2 = [0 0 1 1 1 1 0 0];
```

Now convolve the two using the `conv` function and plot the result using `stem`.

(b) Now create two new vectors as follows:

```
x3 = [1 1 1 1 1 0 0 0];
x2 = [0 0 1 1 1 1 1 0];
```

Again convolve the two using the `conv` function and plot the result using `stem`.

(c) Remember convolution in the time domain is equivalent to multiplication in the Fourier (frequency) domain. So now calculate the convolution from part (a) by taking the FFT of both vectors (using the `fft` function), multiplying the FFTs (using `.*`), and inverse transforming back to the time domain (using the `ifft` function). You may need to take the `real` part of the inverse transform since very small numerical errors will accumulate in MATLAB during the FFT and IFFT process. How does your result compare with part (a)?

(d) Repeat this procedure for the vectors in part (b). How does your result compare with part (b)? Explain what appears to be happening.
Hint 1: We've seen a similar phenomenon earlier in the semester.
Hint 2: What is the FFT/DFT doing in the frequency domain?

Problem 8: MAS 510 Additional Problem

Start in MATLAB with the following commands:

```
n = 0:31;
x = cos(2*pi*0.11*n);
X = abs(fft(x));
```

(a) Explain what the last line accomplishes. Plot `x` and `X` using `stem`. What information does the plot of `X` convey? Does it look like what you'd expect?

(b) Now zero-pad `x` to a length of 2048 using the following command:

```
x1 = [x zeros(1,2016)];
```

Again take the FFT and plot the magnitude of the result. Explain what is going on.

(c) Now window `x` with a Hanning window and zero-pad to a length of 2048 using the following command:

```
x2 = [x.*hanning(32)' zeros(1,2016)];
```

Take the FFT of `x2` and plot the magnitude of the result. Compare this to your plot from part(b). How is it similar and how is it different? Explain what is going on.