Student Name: _____

MAS 622J/1.126J: Pattern Recognition and Analysis

Midterm Quiz, November 1, 2006

This quiz consists of 3 problems, 8 pages. Please make sure you have all 8 pages.

1. (40 points)

   Consider a two-class recognition problem, where your training dataset for $\mathbf{x} = [\, x_1 \; x_2 \,]^T$ indicates that both classes are equally likely and can be modeled by Gaussian distributions with covariances:

   $$\Sigma_1 = \Sigma_2 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

   See the figure on the next page for the one-sigma boundaries for the two classes.

   (a) Label the two axes in the figure.

   (b) Let the mean of class ω_1 be $\mu_1 = [\, 0 \; 0 \,]^T$. What is the mean of class ω_2?

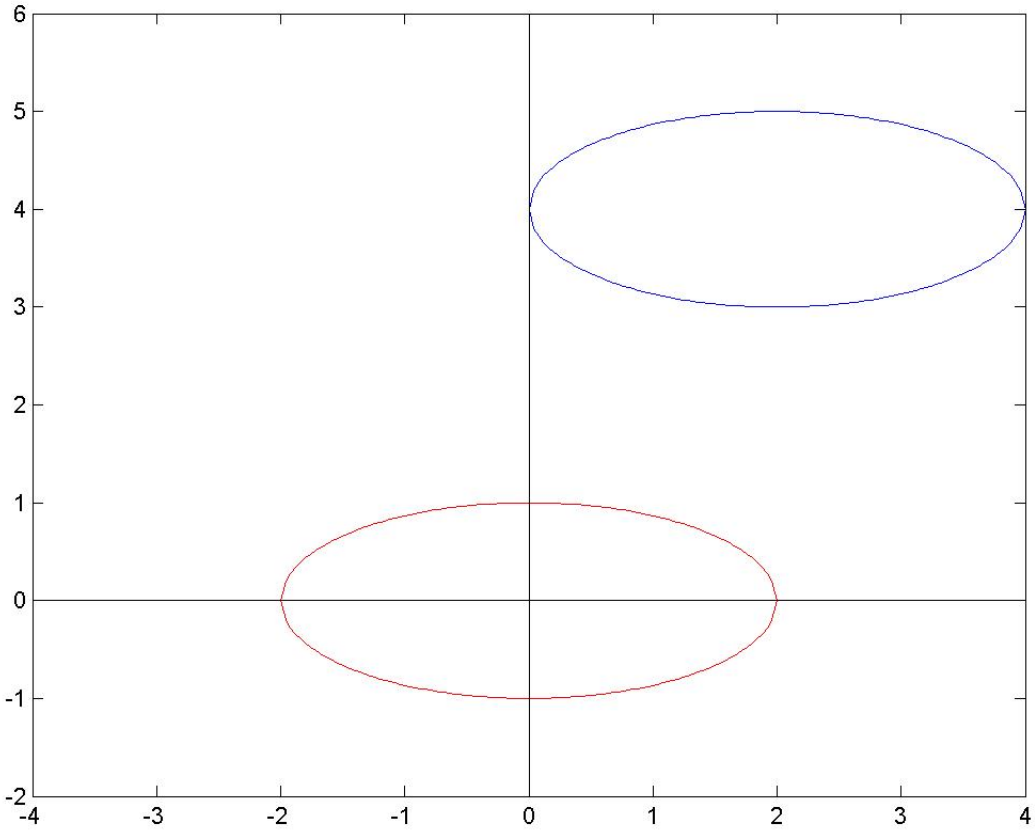
   (c) Consider the criterion of minimizing the Bayes probability of error. Find a mathematical expression for the decision boundary for an unlabeled observation $\mathbf{x}$ that optimizes this criterion.

   (d) Draw the above decision boundary on the plot. If you cannot find the exact formula, then at least specify the type of shape of the boundary (line, ellipse, parabola, hyperbola.)

   (e) Suppose you learn that ω_2 is actually twice as likely to occur as ω_1. Describe how the boundary you found above needs to be changed.

   (f) Consider the criterion of Principle Components Analysis (PCA), where the default PCA method keeps the dimension(s) corresponding to the largest associated eigenvalue(s), and discard the dimension(s) corresponding to the smallest associated eigenvalue(s). Which dimension would be kept here? Would that be the best one to keep for classifying this particular data set? (You can assume equal priors, as originally stated above.)

   (g) Independent of your answer to the previous problem, suppose that you kept the vertical dimension. Without computing the actual error, which method do you think would give the most accurate separation for the two classes: the Bayes method above, or the method of first projecting the data onto the vertical dimension, to "separate" the distributions using only one feature. Explain your reasoning.


6
5
4
3
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4

Please show all your work for partial credit; don't forget to check that you answered all parts of the problem.

(a) (30 points) Joe has plans to take Sue and other friends to a cornfield maze on Saturday. He would really like to ask Sue out to dinner afterward but he is nervous she will say no. He is starting to feel ill thinking about it, and decides to develop a belief net to analyze his chances. Let

a_1 = feels well, a_2 = feels ill

b_1 = navigates the maze well, b_2 = navigates the maze poorly

c_1 = is fun to be with, c_2 = is not fun to be with

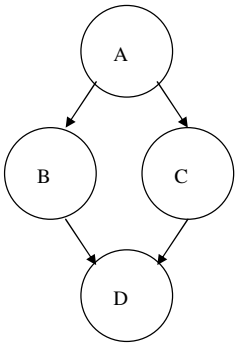
d_1 = she says yes to dinner, d_2 = she says no to dinner

(b) What is the probability, $P(c_1, d_1)$, that Joe will be fun to be with and Sue will say yes to dinner?

(c) Suppose Joe decides that two other things – how much sleep he gets, and how nice the weather is – will both influence how well he will feel. Sketch how you would modify the Bayes Net to accommodate these changes. You do not need to provide probabilities.

(d) (Optional question - answering or not answering will not affect your score) Would you say yes to having dinner with somebody who constructed a Bayes Net to help them think through such a situation?

$P(a_1)$	$P(a_2)$
0.8	0.2

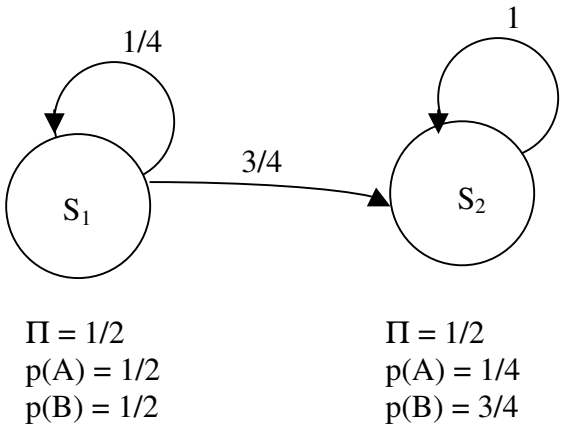


	$P(b_1 \mid a_i)$	$P(b_2 \mid a_i)$
a_1	0.8	0.2
a_2	0.6	0.4

	$P(c_1 \mid a_i)$	$P(c_2 \mid a_i)$
a_1	0.5	0.5
a_2	0.7	0.3

	$P(d_1 \mid b_i, c_i)$	$P(d_2 \mid b_i, c_i)$
b_1, c_1	0.8	0.2
b_1, c_2	0.6	0.4
b_2, c_1	0.4	0.6
b_2, c_2	0.1	0.9

2. (30 points) Suppose you have a two-state HMM defined by the parameters given below.



(a) Find the likelihood this HMM generated the observation sequence: $\mathbf{A}, \mathbf{A}, \mathbf{A}$.

(b) What is the most likely sequence of states that led to these observations?

Please show all your work for partial credit; please check that you answered all parts of each problem.

Page 8 is an extra page in case you need more room to show your work.