Solution to Problem 2

October 17, 2008

1. We use the forward algorithm.

   $\alpha_1(1) = \pi_1 \cdot b_1(B) = 0.75 \times 0.2 = 0.15$

   $\alpha_1(2) = \pi_2 \cdot b_2(B) = 0.25 \times 0.7 = 0.175$

   $\alpha_2(1) = (\alpha_1(1) \cdot a_{11} + \alpha_1(2) \cdot a_{21}) \cdot b_1(A) = (0.15 \cdot 0.7 + 0.175 \cdot 0) \cdot 0.8 = 0.084$

   $\alpha_2(2) = (\alpha_1(1) \cdot a_{12} + \alpha_1(2) \cdot a_{22}) \cdot b_2(A) = (0.15 \cdot 0.3 + 0.175 \cdot 1) \cdot 0.3 = 0.066$

   $\alpha_3(1) = (\alpha_2(1) \cdot a_{11} + \alpha_2(2) \cdot a_{21}) \cdot b_1(B) = (0.084 \cdot 0.7 + 0.066 \cdot 0) \cdot 0.2 = 0.01176$

   $\alpha_3(2) = (\alpha_2(1) \cdot a_{12} + \alpha_2(2) \cdot a_{22}) \cdot b_2(B) = (0.084 \cdot 0.3 + 0.066 \cdot 1) \cdot 0.7 = 0.06384$

   $P(BAB|\lambda) = \alpha_3(1) + \alpha_3(2) = 0.0756$

   Note that if you use the backward algorithm, you will obtain the same answer.

2. We use the Viterbi algorithm

   $\delta_1(1) = \pi_1 \cdot b_1(B) = 0.75 \cdot 0.2 = 0.15$

   $\delta_1(2) = \pi_2 \cdot b_2(B) = 025 \cdot 0.7 = 0.175$

   $\psi_1(1) = 0$

   $\psi_1(2) = 0$

   $\delta_2(1) = \max\{\delta_1(1) \cdot a_{11} \cdot b_1(A), \delta_1(2) \cdot a_{21} \cdot b_1(A)\} = \max\{0.15 \cdot 0.7 \cdot 0.8, 0.175 \cdot 0 \cdot 0.8\} = 0.084$

   $\delta_2(2) = \max\{\delta_1(1) \cdot a_{12} \cdot b_2(A), \delta_1(2) \cdot a_{22} \cdot b_2(A)\} = \max\{0.15 \cdot 0.3 \cdot 0.3, 0.175 \cdot 1 \cdot 0.3\} = 0.0525$

   $\psi_2(1) = 1$, since $\delta_1(1) \cdot a_{11} > \delta_1(2) \cdot a_{21}$.

   $\psi_2(2) = 2$, since $\delta_1(1) \cdot a_{12} < \delta_1(2) \cdot a_{22}$.

   $\delta_3(1) = \max\{\delta_2(1) \cdot a_{11} \cdot b_1(B), \delta_2(2) \cdot a_{21} \cdot b_1(B)\} = \max\{0.084 \cdot 0.7 \cdot 0.2, 0.0525 \cdot 0 \cdot 0.2\} = 0.01176$

   $\delta_3(2) = \max\{\delta_2(1) \cdot a_{12} \cdot b_2(B), \delta_2(2) \cdot a_{22} \cdot b_2(B)\} = \max\{0.084 \cdot 0.3 \cdot 0.7, 0.525 \cdot 1 \cdot 0.7\} = 0.03675$

   $\psi_3(1) = 1$, since $\delta_2(1) \cdot a_{11} > \delta_2(2) \cdot a_{21}$.

   $\psi_3(2) = 2$, since $\delta_2(1) \cdot a_{12} < \delta_2(2) \cdot a_{22}$.

   The optimal sequence of states is therefore 2-2-2.

3. We repeat the evaluation problem we did before, with different model parameters, and select the one with a larger probability value. We still use the forward algorithm.

   $\alpha_1(1) = \pi_1 \cdot b_1(B) = 0.4 \times 0.7 = 0.28$

   $\alpha_1(2) = \pi_2 \cdot b_2(B) = 0.6 \times 0.9 = 0.54$

   $\alpha_2(1) = (\alpha_1(1) \cdot a_{11} + \alpha_1(2) \cdot a_{21}) \cdot b_1(A) = (0.28 \cdot 0.6 + 0.54 \cdot 0) \cdot 0.3 = 0.0504$

   $\alpha_2(2) = (\alpha_1(1) \cdot a_{12} + \alpha_1(2) \cdot a_{22}) \cdot b_2(A) = (0.28 \cdot 0.4 + 0.54 \cdot 1) \cdot 0.1 = 0.0652$

   $\alpha_3(1) = (\alpha_2(1) \cdot a_{11} + \alpha_2(2) \cdot a_{21}) \cdot b_1(B) = (0.0504 \cdot 0.6 + 0.0652 \cdot 0) \cdot 0.7 = 0.02117$

   $\alpha_3(2) = (\alpha_2(1) \cdot a_{12} + \alpha_2(2) \cdot a_{22}) \cdot b_2(B) = (0.0504 \cdot 0.4 + 0.0652 \cdot 1) \cdot 0.9 = 0.07682$

   $P(BAB|\lambda) = \alpha_3(1) + \alpha_3(2) = 0.098 > 0.0756$

   The conclusion is that model 2 better explains the observed data.