Equivalent ways to represent the system

1. \[ y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k] \] difference equation

2. \[ x[n], h[n], a_i, b_i, a, b \] block diagram

3. \[ h[n] = y[n] \bigg|_{x[n] = \delta[n]} \] impulse response

4. \[ H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{\prod_{i=0}^{M} (z - z_{zi})}{\prod_{i=0}^{N} (z - z_{pi})} \] system function polynomial

5. \[ z = e^{j\omega} \] pole-zero locations

6. \[ H(\omega) = H(e^{j\omega}) = H(z) \bigg|_{z = e^{j\omega}} \] frequency response

All poles must be inside unit circle for \( H(\omega) \) to converge and the system to be stable. (FIR filter always stable)
Q: What is the definition of an FIR filter?

A: The output $y$ at each sample $n$ is a weighted sum of the present input, $x[n]$, and past inputs, $x[n-1], x[n-2], \ldots, x[n-M]$. 
Q: What is the definition of an FIR filter?
A: The output $y$ at each sample $n$ is a weighted sum of the present input, $x[n]$, and past inputs, $x[n-1], x[n-2], \ldots, x[n-M]$. 
Causal FIR filter

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]

\[ y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M] \]

The output \( y \) at each sample \( n \) is a weighted sum of the present input, \( x[n] \), and past inputs, \( x[n-1], x[n-2], \ldots, x[n-M] \).
3 point average

\[ y[n] = \sum_{k=0}^{M} b_k x[n - k] \]

\( L = 3 \) \quad \text{Length 3} \quad \text{M=L-1=2} \quad \text{2nd order} \quad b_0 = \frac{1}{3} \quad b_1 = \frac{1}{3} \quad b_2 = \frac{1}{3} \]

\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n - 1] + \frac{1}{3} x[n - 2] \]
3 point average

\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0 \ 0\} \]

\[ n=-2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0, 0, 1.11, 1.16, 1.01, 1.12, 1.01, 1.08, 0, 0\} \]

\[ n = -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 \]

\[ y[0] = \frac{1}{3} x[0] + \frac{1}{3} x[-1] + \frac{1}{3} x[-2] = \frac{1}{3} 1.11 + \frac{1}{3} 0 + \frac{1}{3} 0 = 0.36 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.01 \ 1.08 \ 0 \ 0\} \]

\[ n = \{-2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\} \]

\[ y[1] = \frac{1}{3} x[1] + \frac{1}{3} x[0] + \frac{1}{3} x[-1] = \frac{1}{3} 1.16 + \frac{1}{3} 1.11 + \frac{1}{3} 0 = 0.76 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.01 \ 1.08 \ 0 \ 0\} \]

\[ n = -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \]

\[ y[2] = \frac{1}{3} x[2] + \frac{1}{3} x[1] + \frac{1}{3} x[0] = \frac{1}{3} \cdot 1.01 + \frac{1}{3} \cdot 1.16 + \frac{1}{3} \cdot 1.11 = 1.09 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\} \]

\[ n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \]

\[ y[3] = \frac{1}{3} x[3] + \frac{1}{3} x[2] + \frac{1}{3} x[1] = \frac{1}{3} 1.12 + \frac{1}{3} 1.01 + \frac{1}{3} 1.16 = 1.10 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]
\[ x[n] = \{ 0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.08 \ 0 \ 0 \} \]
\[ n = -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \]
\[ y[4] = \frac{1}{3} x[4] + \frac{1}{3} x[3] + \frac{1}{3} x[2] = \frac{1}{3} \cdot 1.01 + \frac{1}{3} \cdot 1.12 + \frac{1}{3} \cdot 1.01 = 1.05 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.08 \ 0 \ 0\} \]

\[ y[5] = \frac{1}{3} x[5] + \frac{1}{3} x[4] + \frac{1}{3} x[3] = \frac{1}{3} 1.08 + \frac{1}{3} 1.01 + \frac{1}{3} 1.12 = 1.07 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0, 0, 1.11, 1.16, 1.01, 1.12, 1.01, 1.08, 0, 0\} \]

\[ n = -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 \]

\[ y[6] = \frac{1}{3} x[6] + \frac{1}{3} x[5] + \frac{1}{3} x[4] = \frac{1}{3} 0 + \frac{1}{3} 1.08 + \frac{1}{3} 1.01 = 0.70 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0 \ 0\} \]

\[ n = -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \]

\[ y[7] = \frac{1}{3} x[7] + \frac{1}{3} x[6] + \frac{1}{3} x[5] = \frac{1}{3} \times 1.0 + \frac{1}{3} \times 1.0 + \frac{1}{3} \times 1.08 = 0.36 \]
\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \{0, 0, 1.11, 1.16, 1.01, 1.12, 1.01, 1.08, 0.0, 0.0\} \]

\[ y[n] = \{0, 0, 0.36, 0.75, 1.09, 1.10, 1.05, 1.07, 0.7, 0.36\} \]

\[ n = -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 \]
2 point difference

\[ y[n] = x[n] - x[n-1] \]

\[ x[n] = \frac{n^2}{16} \cdot u[n] \]

\[ u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \]

unit step function
2 point difference

\[ y[n] = x[n] - x[n-1] \]

\[ x[n] = \frac{n^2}{16} \cdot u[n] \]

\[ u[n] = \begin{cases} 
1 & n \geq 0 \\
0 & n < 0 
\end{cases} \]

unit step function
\[ y[n] = x[n] - x[n-1] \]
\[ x[n] = \frac{n^2}{16} \cdot u[n] \]

finite difference approximation to a derivative.

\[ x[n] = \{0 \ 0 \ 0 \ \frac{1}{16} \ \frac{4}{16} \ \frac{9}{16} \ \frac{16}{16} \} \]

derivatives enhance noise (and high frequencies)

\[ y[n] = \{0 \ 0 \ 0 \ \frac{1}{16} \ \frac{3}{16} \ \frac{5}{16} \ \frac{7}{16} \} = \frac{2n-1}{16} u[n-1] \]
**Impulse response**

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] \quad \text{FIR filter}$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Delta function}$$

$$y[n] = h[n] = \sum_{k=0}^{M} b_k \delta[n-k] \quad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & \text{otherwise} \end{cases} \quad \text{impulse response}$$
Impulse response

\[ h[n] = y[n] = \sum_{k=0}^{M} b_k \delta[n - k] \]

\[ \delta[n - k] = \begin{cases} 1 & n = k \\ 0 & \text{otherwise} \end{cases} \]

\[ h[n] = b_0 \delta[n - 0] + b_1 \delta[n - 1] + \ldots + b_n \delta[n - n] + \ldots + b_M \delta[M - 1] \]

\[ = b_0 \delta[n - 0] + b_1 \delta[n - 1] + \ldots + b_n \delta[0] + \ldots + b_M \delta[M - 1] \]

\[ = b_0 0 + b_1 0 + \ldots + b_n 1 + \ldots + b_M 0 \]

\[ = b_n \quad \text{impulse response} \]

The impulse response is just the filter coefficients.

Finite length filter, finite impulse response (FIR).
Impulse response of 3 pt. average

\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \delta[n] = \begin{cases} 
1 & n = 0 \\
0 & \text{otherwise}
\end{cases} \]  \hspace{1cm} \text{Delta function}

\[ y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2] \]

\[ y[0] = \frac{1}{3} \delta[0] + \frac{1}{3} \delta[-1] + \frac{1}{3} \delta[-2] \]

\[ = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \]
Impulse response of 3 pt. average

\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \delta[n] = \begin{cases} 
1 & n = 0 \\
0 & \text{otherwise} 
\end{cases} \]

\[ y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2] \]

\[ y[1] = \frac{1}{3} \delta[1] + \frac{1}{3} \delta[0] + \frac{1}{3} \delta[-1] = \frac{1}{3}0 + \frac{1}{3}1 + \frac{1}{3}0 = \frac{1}{3} \]
Impulse response of 3 pt. average

\[
y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]
\]

\[
x[n] = \delta[n] = \begin{cases} 
1 & n = 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2]
\]

\[
y[2] = \frac{1}{3} \delta[2] + \frac{1}{3} \delta[1] + \frac{1}{3} \delta[0]
\]

\[
= \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 1 = \frac{1}{3}
\]
Impulse response of 3 pt. average

\[ y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \]

\[ x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \]

\[ y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2] \]

\[ y[3] = \frac{1}{3} \delta[3] + \frac{1}{3} \delta[2] + \frac{1}{3} \delta[1] \]

\[ = \frac{1}{3} 0 + \frac{1}{3} 0 + \frac{1}{3} 0 = 0 \]
Impulse response of 3 pt. average

\[ n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\} \]

\[ h[n] = y[n] |_{x[n]=\delta[n]} = \{0 \quad 0 \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad 0 \quad 0\} \]

\[ = \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\} \]
Coefficients from impulse response

\[ x[n] = \delta[n] \]
\[ = \{0, 0, 1, 0, 0, 0, 0\} \]
\[ n = -2, -1, 0, 1, 2, 3, 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n - k] \]
\[ b_0, b_1, \ldots, b_M = ?? \]

\[ h[n] = y[n] \bigg|_{x[n] = \delta[n]} \]
\[ = \{0, 0, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}, 0, 0\} \]
Coefficients from impulse response

\[ x[n] = \delta[n] \]
\[ = \{0, 0, 1, 0, 0, 0, 0\} \]
\[ n = -2, -1, 0, 1, 2, 3, 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n - k] \]
\[ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \]

\[ h[n] = y[n] \big|_{x[n] = \delta[n]} \]
\[ = \{0, 0, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}, 0, 0\} \]
\[ = \{0, 0, b_0, b_1, b_2, 0, 0\} \]
Response from 2 impulses

\[ x[n] = \delta[n] = \{0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0\} \]

\[ n = -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]

\[ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \]

\[ y[n] = \{0 \ 0 \ \frac{4}{8} \ \frac{2}{8} \ \frac{1}{8} + \frac{4}{16} \ \frac{2}{8} \ \frac{1}{16}\} \]


Sum the responses of each impulse
\[ x[n] = n^2 \cdot u[n] \]

\[ u[n] = \begin{cases} 
1 & n \geq 0 \\
0 & n < 0 
\end{cases} \]

\[ x[n] = \{0, 0, 0, \frac{1}{16}, \frac{4}{16}, \frac{9}{16}, \frac{16}{16}\} \quad n = \{-2, -1, 0, 1, 2, 3, 4\} \]

\[ x[n] = 0 \cdot \delta[n] + \frac{1}{16} \cdot \delta[n-1] + \frac{4}{16} \cdot \delta[n-2] + \frac{9}{16} \cdot \delta[n-3] + \frac{16}{16} \cdot \delta[n-4] \]

Any discrete signal be thought of a weighted sum of delayed impulses.
Response from 2 impulses

\[ x[n] = \delta[n] \]
\[ = \{0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0\} \]
\[ n = -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]
\[ h[n] = \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \]

\[ y[n] = \{0 \ 0 \ \frac{4}{8} \ \frac{2}{8} \ \frac{1}{8} + \frac{4}{16} \ \frac{2}{8} \ \frac{1}{16}\} \]


\[ + \ h[1]x[2 - 1] \]
\[ + \ h[2]x[2 - 2] \]

\[ y[n] = \sum_{k=0}^{3} h[k]x[n-k] \]

Convolution sum
LTI Systems

\[ y[n] = h[n] \cdot x[n] \]

\[ y[n] = h_1[n] \cdot h_2[n] \cdot x[n] \]

\[ y[n] = (h_1[n] + h_2[n]) \cdot x[n] \]
LTI Systems

\[ y[n] = h[n] \times x[n] \]

\[ y[n] = h_1[n] \times h_2[n] \times x[n] \]

\[ y[n] = (h_1[n] + h_2[n]) \times x[n] \]
LTI Systems
LTI Systems

\[ x[n] \rightarrow h_1[n] + h_2[n] \rightarrow h_3[n] \rightarrow y[n] \]
LTI Systems

\[ x[n] \rightarrow h_3 \ast (h_1[n] + h_2[n]) \rightarrow y[n] \]
Block Diagrams

1. $x[n] \rightarrow \text{unit delay} \rightarrow y[n] = x[n-1]

2. $x[n] \rightarrow A \rightarrow y[n] = Ax[n]

3. $x[n] \rightarrow + \rightarrow y[n] = x[n] + z[n]$

4. $z[n] \rightarrow + \rightarrow y[n] = z[n]$
Block Diagrams: Direct Form

\[ x[n] = \delta[n] \]
\[ x[n] = \{0, 0, 1, 0, 0, 0, 0\} \]
\[ n = -2, -1, 0, 1, 2, 3, 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]
\[ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \]
\[ L=3, M=L-1=2 \]

\[ h[n] = y[n]_{x[n]=\delta[n]} \]
\[ h[n] = \{0, 0, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}, 0, 0\} \]
\[ = \{0, 0, b_0, b_1, b_2, 0, 0\} \]

Graphs:
- \( x[n] \)
- \( h[n] \)
Block Diagrams: Direct Form

\[ x[n] = \delta[n] \]
\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]
\[ \{b_0, b_1, b_2\} = \left\{ \frac{4}{8}, \frac{2}{8}, \frac{1}{8} \right\} \]
\[ h[n] = y[n] \bigg|_{x[n] = \delta[n]} \]

\[ y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \]

L=3, M=L-1=2

\[ \{0, 0, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}, 0, 0\} \]

\[ \{0, 0, b_0, b_1, b_2, 0, 0\} \]
**Block Diagrams: Transpose Form**

\[ x[n] = \delta[n] \]
\[ = \{0, 0, 1, 0, 0, 0, 0\} \]
\[ n = -2, -1, 0, 1, 2, 3, 4 \]

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]
\[ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \]

\[ h[n] = y[n] \bigg|_{x[n]=\delta[n]} \]
\[ = \{0, 0, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}, 0, 0\} \]
\[ = \{0, 0, b_0, b_1, b_2, 0, 0\} \]

\[ y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \]

\[ b_2 x[n] \]
\[ b_1 x[n] \]
\[ b_0 x[n] \]

\[ b_2 x[n-1] \]
\[ b_2 x[n-2] + b_1 x[n-1] \]

\[ y[n] \]
Block Diagrams to Difference Equations

\[ x[n] \]

\[ v_2[n] = b_2 x[n] \]

\[ v_1[n] = b_1 x[n] + v_2[n-1] \]

\[ v_1[n] = b_1 x[n] + v_2[n-1] \]

\[ v_2[n] = b_2 x[n] \]

\[ y[n] = b_0 x[n] + v_1[n-1] \]

\[ y[n] = b_0 x[n] + v_1[n-1] \]
Block Diagrams to Difference Equations

\[ x[n] \]

\[ \begin{align*}
v_2[n] &= b_2 x[n] \\
v_1[n] &= b_1 x[n] + v_2[n-1] \\
y[n] &= b_0 x[n] + v_1[n-1]
\end{align*} \]

\[ \begin{align*}
v_2[n-2] &= b_2 x[n-2] \\
v_1[n-1] &= b_1 x[n-1] + v_2[n-2] \\
y[n] &= b_0 x[n] + v_1[n-1]
\end{align*} \]
Block Diagrams to Difference Equations

\[ x[n] \]

\[ v_2[n] = b_2 x[n] \]

\[ v_1[n] = b_1 x[n] + v_2[n - 1] \]

\[ y[n] = b_0 x[n] + v_1[n - 1] \]

\[ v_1[n - 1] = b_1 x[n - 1] + b_2 x[n - 2] \]
Block Diagrams to Difference Equations

\[ x[n] \]

\[ v_2[n] = b_2 x[n] \]

\[ v_1[n] = b_1 x[n] + v_2[n-1] \]

\[ y[n] = b_0 x[n] + v_1[n-1] \]

\[ y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \]
Block Diagrams to Difference Equations

\[ x[n] \]

\[ \begin{align*}
    v_2[n] &= b_2 x[n] \\
    v_1[n] &= b_1 x[n] + v_2[n - 1] \\
    y[n] &= b_0 x[n] + v_1[n - 1]
\end{align*} \]

\[ \begin{align*}
    y[n] &= b_0 x[n] + b_1 x[n - 1] + b_2 x[n - 2] \\
    h[n] &= b_0 \delta[n] + b_1 \delta[n - 1] + b_2 \delta[n - 2]
\end{align*} \]

difference equation
impulse response

equivalent ways of describing system
Block Diagrams to Difference Equations

\[ x[n] \rightarrow v_2[n] \rightarrow \text{unit delay} \rightarrow v_1[n] \rightarrow \text{unit delay} \rightarrow y[n] \]

- \[ v_2[n] = x[n] \]
- \[ v_1[n] = v_2[n-1] \]
- \[ y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1] \]
Block Diagrams to Difference Equations

\[ v_2[n] = x[n] \]
\[ v_1[n] = v_2[n-1] \]
\[ y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1] \]
Block Diagrams to Difference Equations

\[ v_2[n] = x[n] \]
\[ v_1[n] = v_2[n-1] \]
\[ y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1] \]

\[ y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \]
Homework:

\[ y(n) := \frac{1}{L} \left[ \sum_{k=0}^{L-1} a^{n-k} u(n-k) \right] \]

**p5_1:**

\[ y(n) := \frac{1}{L} \left[ \sum_{k=0}^{L-1} a^{n-k} u(n-k) \right] \]

**hint:**

\[ \sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1 - \alpha} \]

\[ \Rightarrow \frac{1}{L} \left[ \sum_{n-z=0}^{L-1} a^z u(z) \right] \]

\[ \Rightarrow \frac{1}{L} \left[ \sum_{z=n}^{n-(L-1)} a^z u(z) \right] \]

**L-point running average**

for input sequence

\[ x[n] = a^n u[n], \quad n \geq 0 \]

remember \( n \geq 0 \)

**p5_6:**

FIR & delays
Homework:

p5_1: \[ y(n) := \frac{1}{L} \left[ \sum_{k=0}^{L-1} a^{n-k} \cdot u(n-k) \right] \]

hint: \[ \sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1 - \alpha} \]

let \( z = n-k \)
\( k = n-z \)

L-point running average for input sequence \( x[n] = a^n u[n], \ n \geq 0 \)

\[ \frac{1}{L} \left[ \sum_{n-z=0}^{L-1} a^z \cdot u(z) \right] \]

\[ \frac{1}{L} \left[ \sum_{z=n}^{n-(L-1)} a^z \cdot u(z) \right] \]

remember \( n \geq 0 \)

p5_6: FIR & delays
FIR and single delay

\[ y[n] = a x[n] + b x[n-1] \]
3 point average

\[
y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \quad x[n] = \sin(2\pi n/15) \cdot u[n] \quad u[n] = \begin{cases} 
1 & n \geq 0 \\
0 & n < 0 
\end{cases}
\]

\[x[n]\]

\[n\]