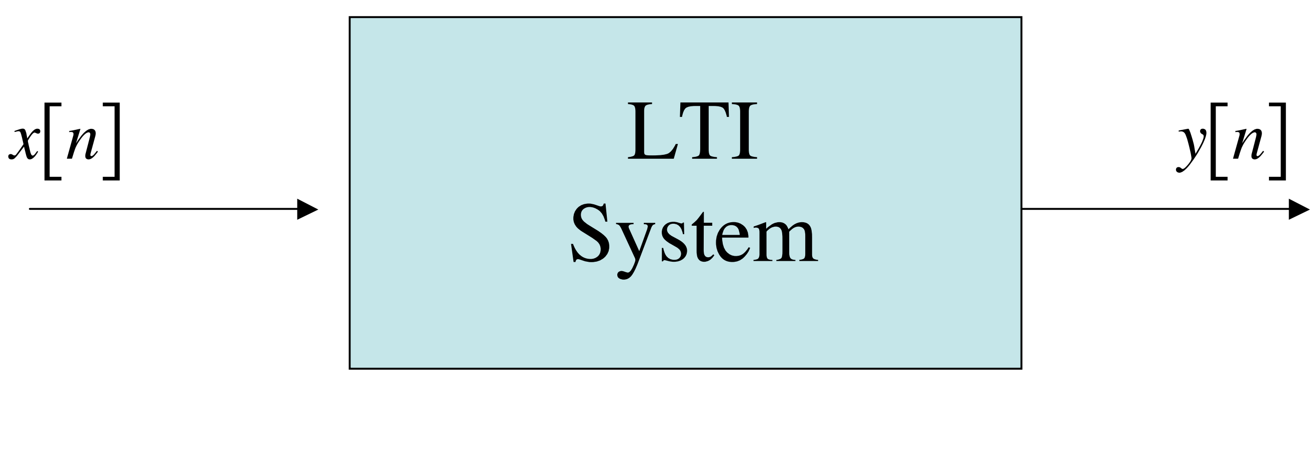


x[n]
LTI
System
y[n]

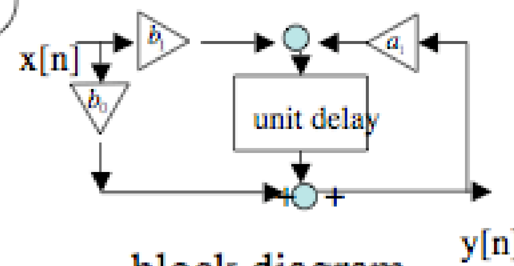
Equivalent ways to represent the system

(1)

$$y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k] \quad \Leftrightarrow$$

difference equation

(2)



block diagram

$x[n]=\delta[n]$ $\Updownarrow$

$\Updownarrow$ inspection

(3)

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} \overset{z}{\Leftrightarrow} H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{\prod_{i=0}^{M}(z - z_{zi})}{\prod_{i=0}^{N}(z - z_{pi})}$$

impulse response sequence

(4) system function polynomial

pole-zero locations (5)

$z = e^{j\omega}$ $\Updownarrow$

(6)

$$\mathcal{H}(\omega) = H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}}$$

frequency response

All poles must be inside unit circle for $\mathcal{H}(\omega)$ to converge and the system to be stable.
(FIR filter always stable)

Causal FIR filter

Q: What is the definition of an FIR filter?

A: The output y at each sample n is a weighted sum of the present input, x[n], and past inputs, x[n-1], x[n-2],…, x[n-M].

Causal FIR filter

Q: What is the definition of an FIR filter?

A: The output y at each sample n is a weighted sum of the present input, x[n], and past inputs, x[n-1], x[n-2],…, x[n-M].

Causal FIR filter

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

The output y at each sample n is a weighted sum of the present input, x[n], and past inputs, x[n-1], x[n-2],…, x[n-M].

3 point average

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

L=3 M=L-1=2

Length 3 2nd order

$$b_0=\frac{1}{3} \quad b_1=\frac{1}{3} \quad b_2=\frac{1}{3}$$

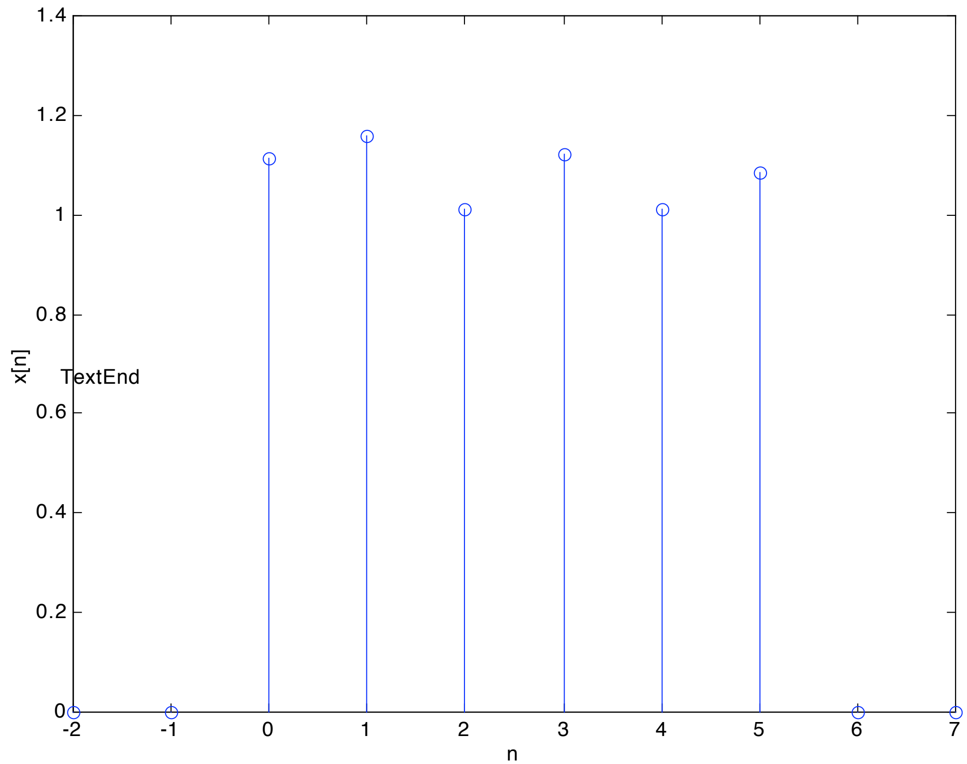
$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

3 point average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n=-2 -1 0 1 2 3 4 5 6 7

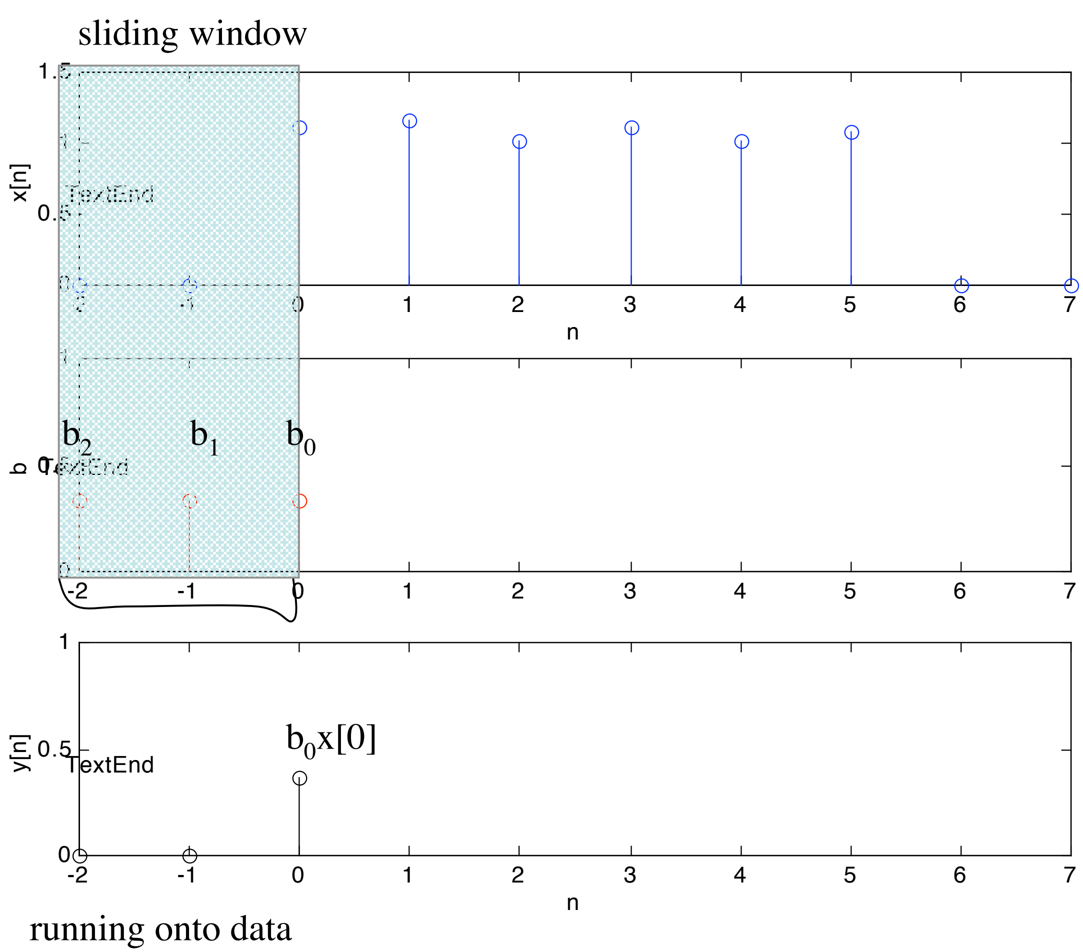


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$$x[n]=\{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[0]=\frac{1}{3}x[0]+\frac{1}{3}x[-1]+\frac{1}{3}x[-2]=\frac{1}{3}1.11+\frac{1}{3}0+\frac{1}{3}0=0.36$$

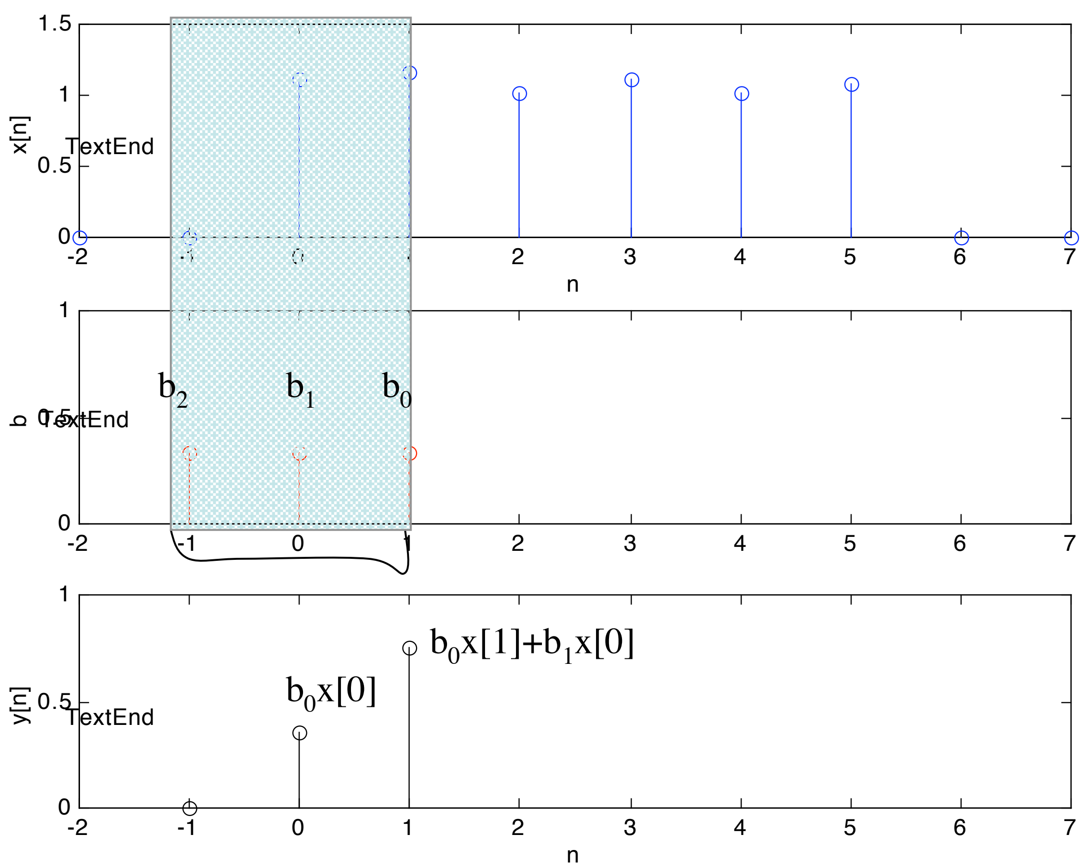


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[1] = \frac{1}{3}x[1] + \frac{1}{3}x[0] + \frac{1}{3}x[-1] = \frac{1}{3}1.16 + \frac{1}{3}1.11 + \frac{1}{3}0 = 0.76$$

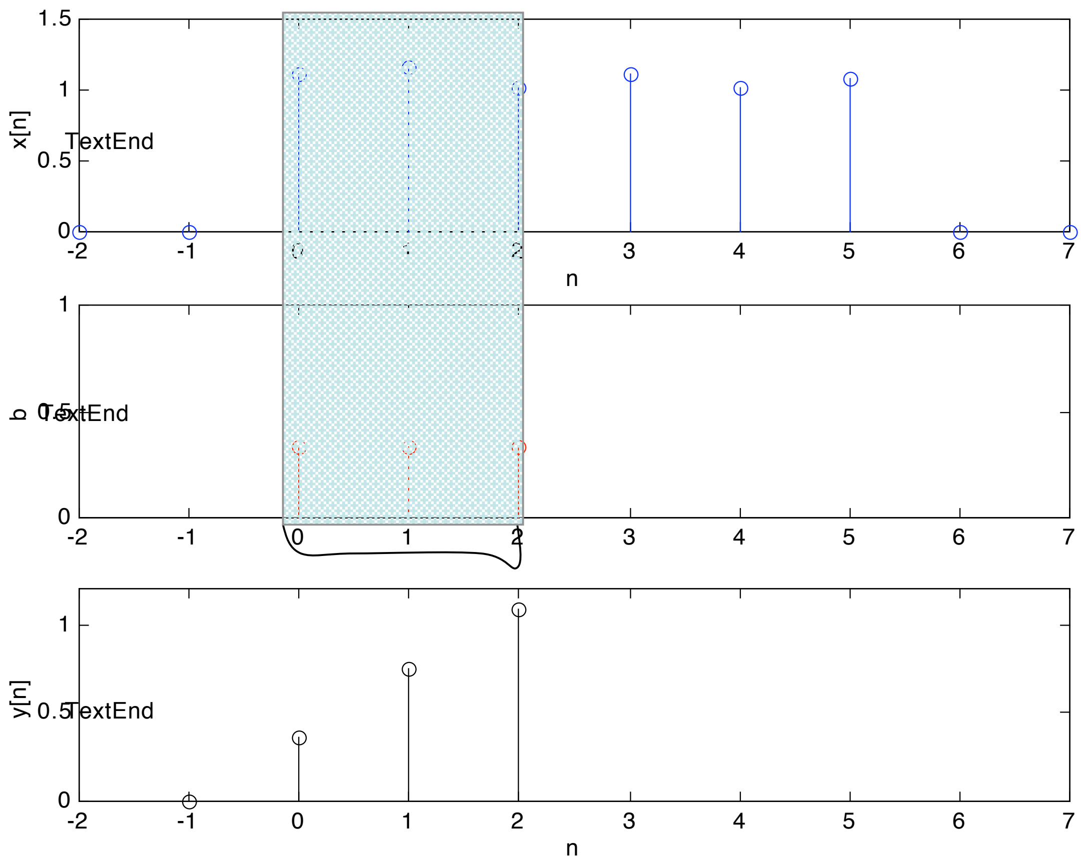


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$$x[n]=\{0\quad 0\quad 1.11\quad 1.16\quad 1.01\quad 1.12\quad 1.01\quad 1.08\quad 0\quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[2]=\frac{1}{3}x[2]+\frac{1}{3}x[1]+\frac{1}{3}x[0]=\frac{1}{3}1.01+\frac{1}{3}1.16+\frac{1}{3}1.11=1.09$$



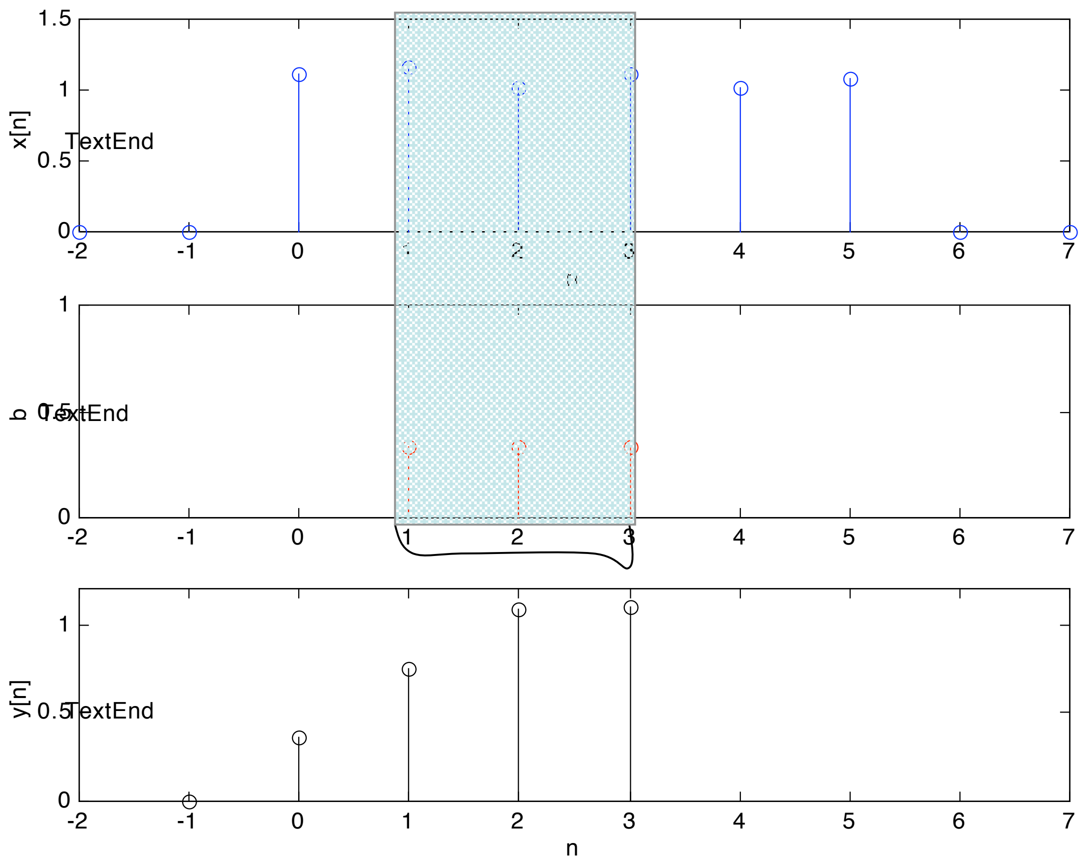
weighted sum

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[3] = \frac{1}{3}x[3] + \frac{1}{3}x[2] + \frac{1}{3}x[1] = \frac{1}{3}1.12 + \frac{1}{3}1.01 + \frac{1}{3}1.16 = 1.10$$

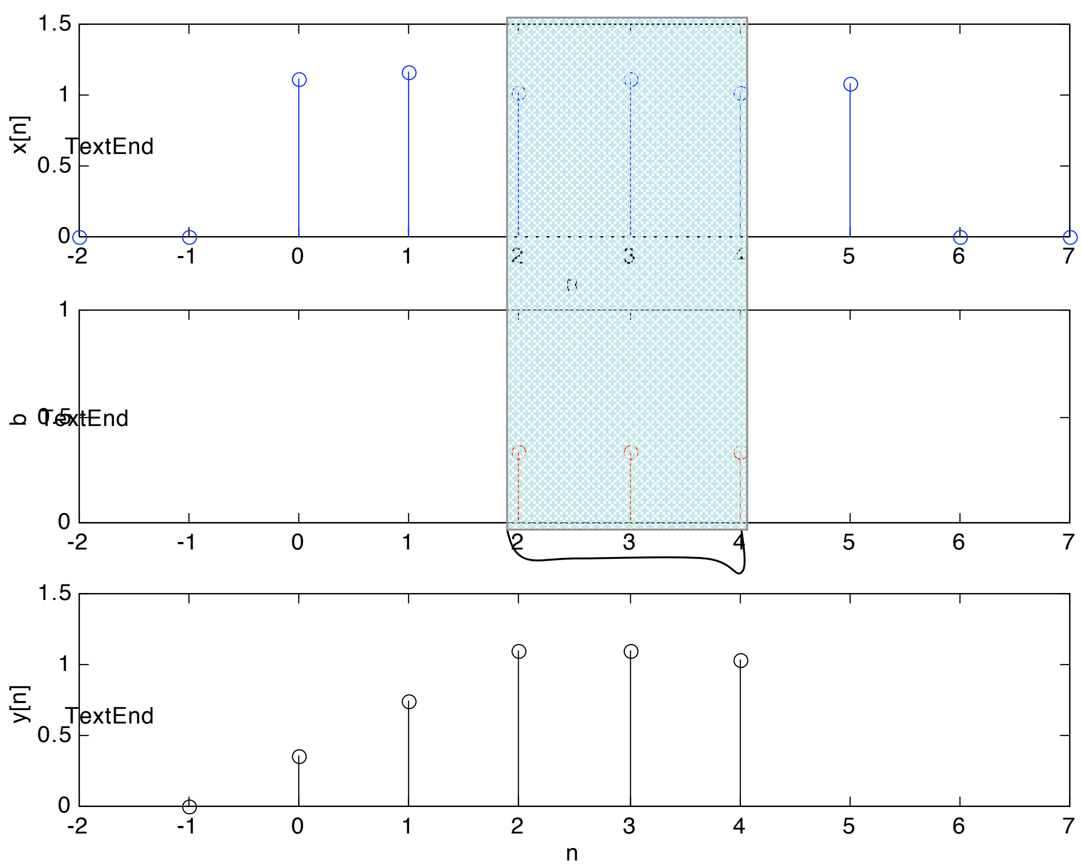


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

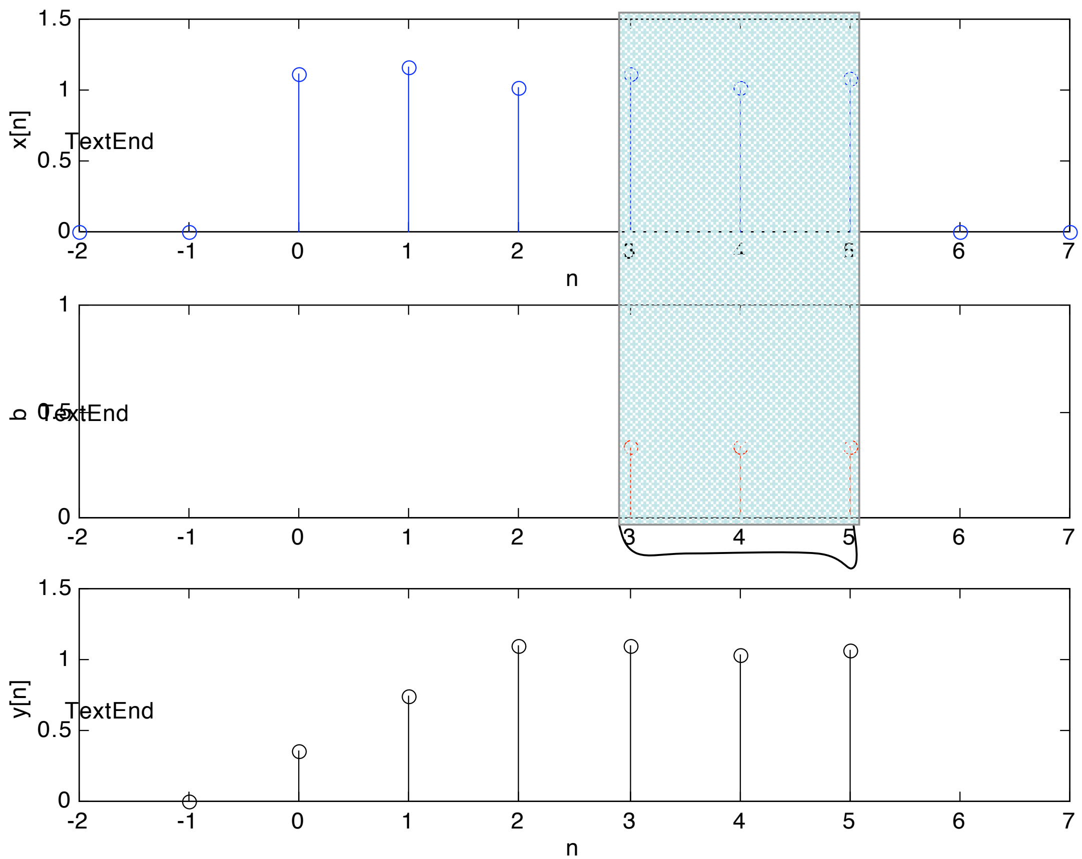
$$y[4] = \frac{1}{3}x[4] + \frac{1}{3}x[3] + \frac{1}{3}x[2] = \frac{1}{3}1.01 + \frac{1}{3}1.12 + \frac{1}{3}1.01 = 1.05$$



$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

$$y[5] = \frac{1}{3}x[5] + \frac{1}{3}x[4] + \frac{1}{3}x[3] = \frac{1}{3}1.08 + \frac{1}{3}1.01 + \frac{1}{3}1.12 = 1.07$$

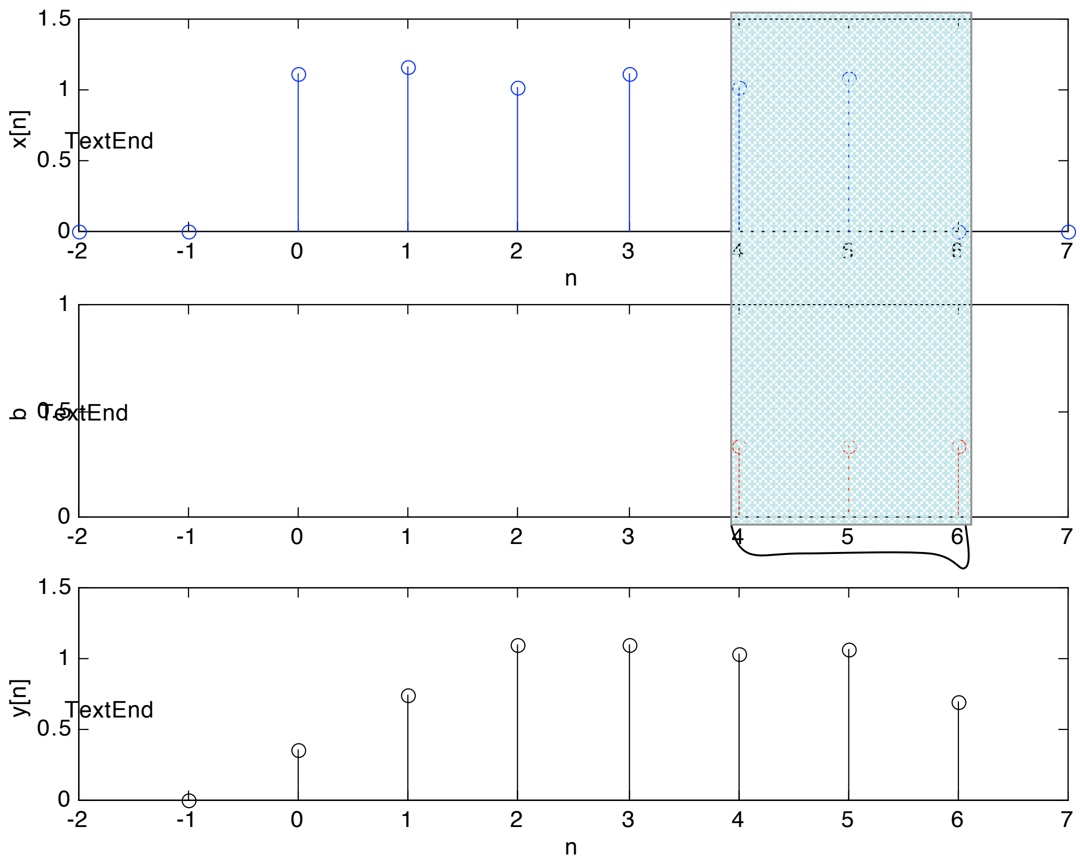


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[6] = \frac{1}{3}x[6] + \frac{1}{3}x[5] + \frac{1}{3}x[4] = \frac{1}{3}0 + \frac{1}{3}1.08 + \frac{1}{3}1.01 = 0.70$$



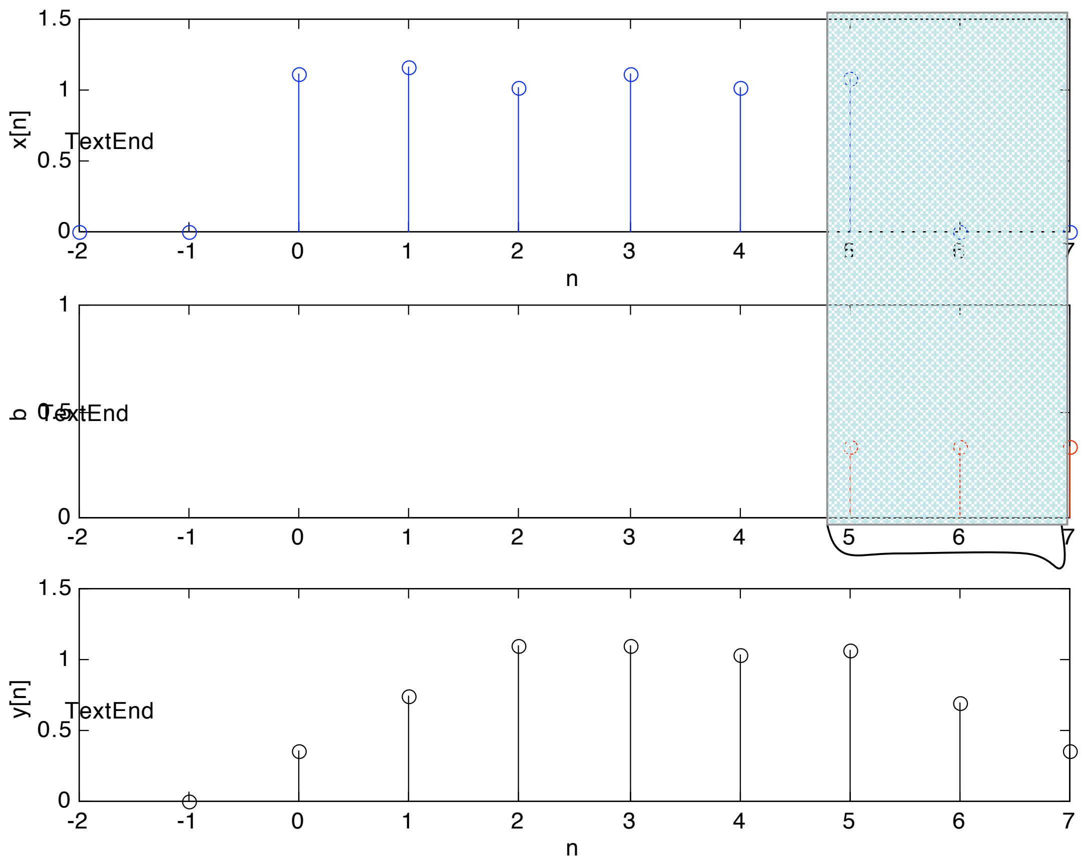
running off data

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[7] = \frac{1}{3}x[7] + \frac{1}{3}x[6] + \frac{1}{3}x[5] = \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}1.08 = 0.36$$



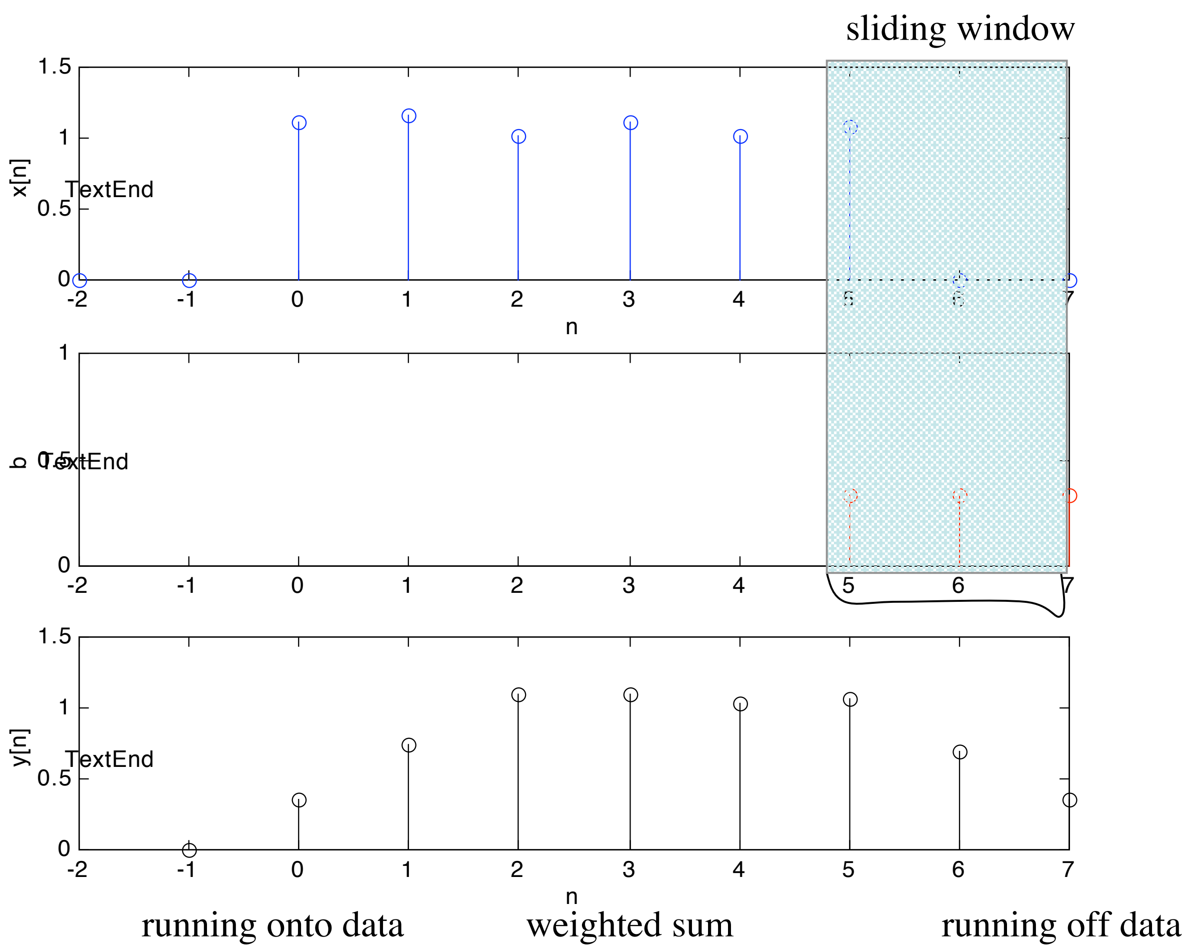
running off data

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0.0 \quad 0.0\}$$

$$y[n] = \{0 \quad 0 \quad 0.36 \quad 0.75 \quad 1.09 \quad 1.10 \quad 1.05 \quad 1.07 \quad 0.7 \quad 0.36\}$$

n= -2 -1 0 1 2 3 4 5 6 7



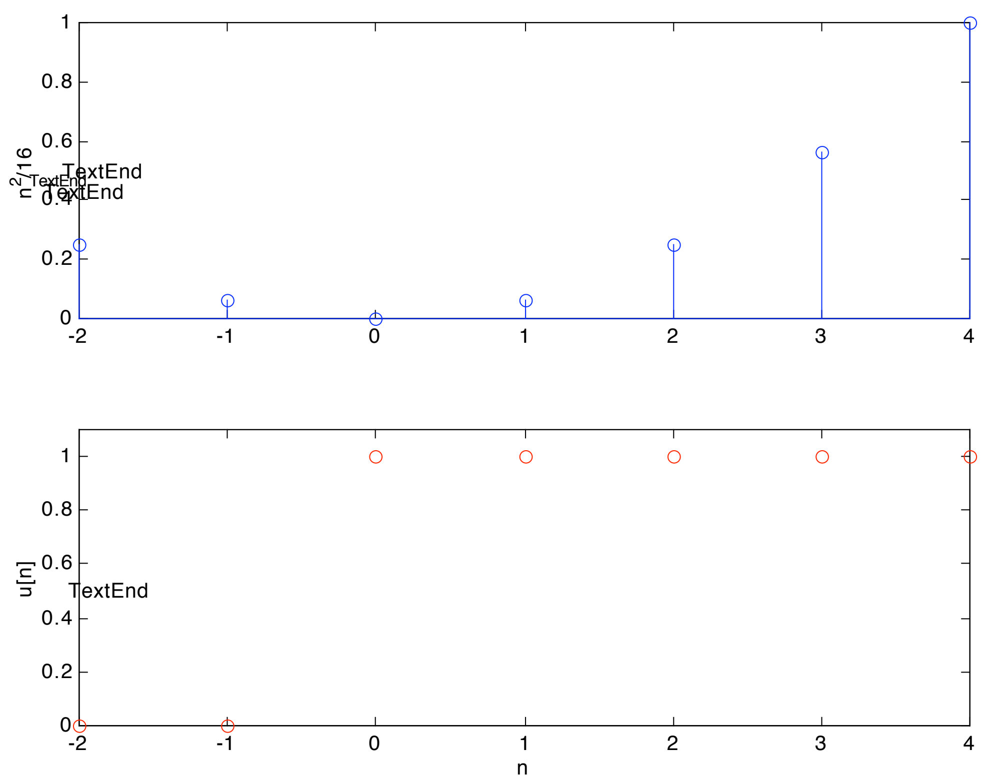
2 point difference

$$y[n] = x[n] - x[n-1]$$

$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit step function



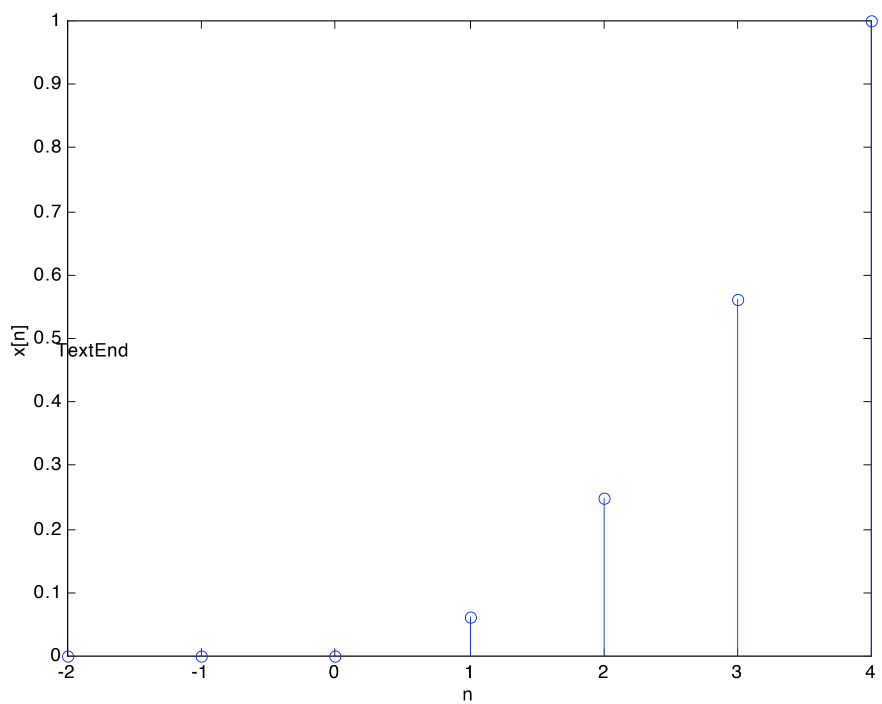
2 point difference

$$y[n] = x[n] - x[n-1]$$

$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit step function



$$y[n] = x[n] - x[n-1]$$

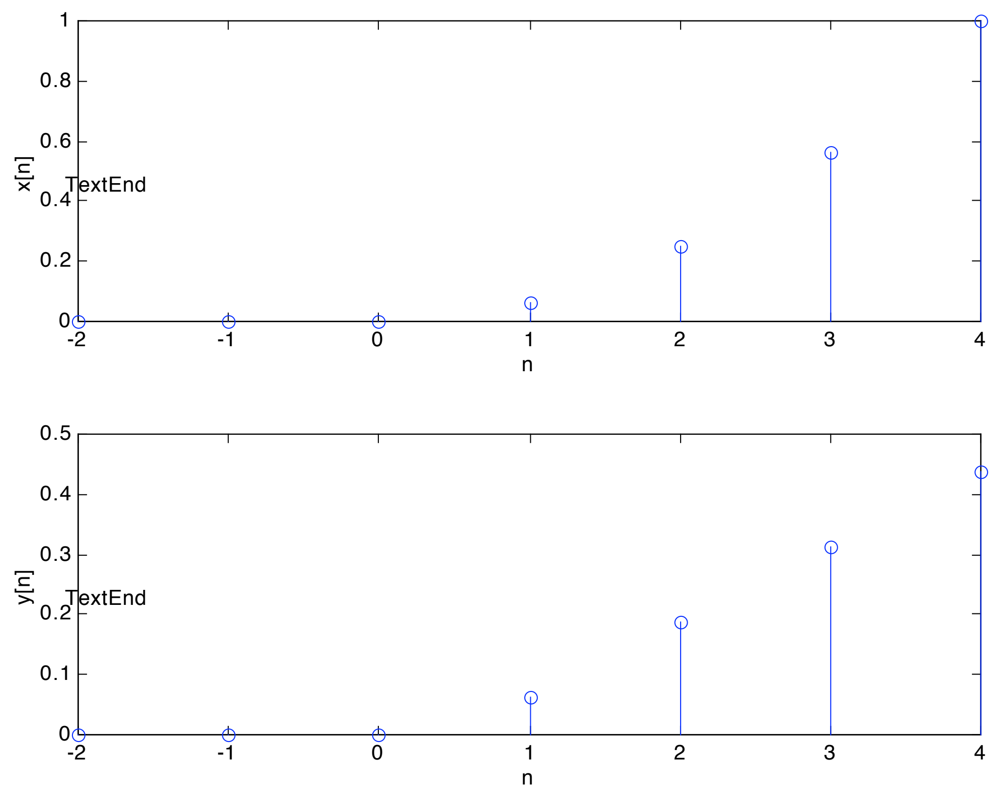
$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$x[n] = \{0 \quad 0 \quad 0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16}\}$$

$$y[n] = \{0 \quad 0 \quad 0 \quad \tfrac{1}{16} \quad \tfrac{3}{16} \quad \tfrac{5}{16} \quad \tfrac{7}{16}\} = \frac{2n-1}{16} u[n-1]$$

finite difference approximation to a derivative.

derivatives enhance noise (and high frequencies)



Impulse response

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$ FIR filter

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$ Delta function

↓

$$y[n] = h[n] = \sum_{k=0}^{M} b_k \delta[n-k] \qquad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & otherwise \end{cases}$$

impulse response

Impulse response

$$h[n] = y[n] = \sum_{k=0}^{M} b_k \delta[n-k] \qquad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & otherwise \end{cases}$$

$$\begin{aligned} h[n] &= b_0\delta[\overset{k=0}{n-0}] + b_1\delta[n-1] + \ldots + b_n\delta[\overset{k=n}{n-n}] + \ldots + b_M\delta[M-1] \\ &= b_0\delta[n-0] + b_1\delta[n-1] + \ldots + b_n\delta[0] + \ldots + b_M\delta[M-1] \\ &= b_0 0 + b_1 0 + \ldots + b_n 1 + \ldots + b_M 0 \\ &= b_n \quad \text{impulse response} \end{aligned}$$

$$\delta[z] = \begin{cases} 1 & z = 0 \\ 0 & otherwise \end{cases}$$

The impulse response is just the filter coefficients.
Finite length filter, finite impulse response (FIR).

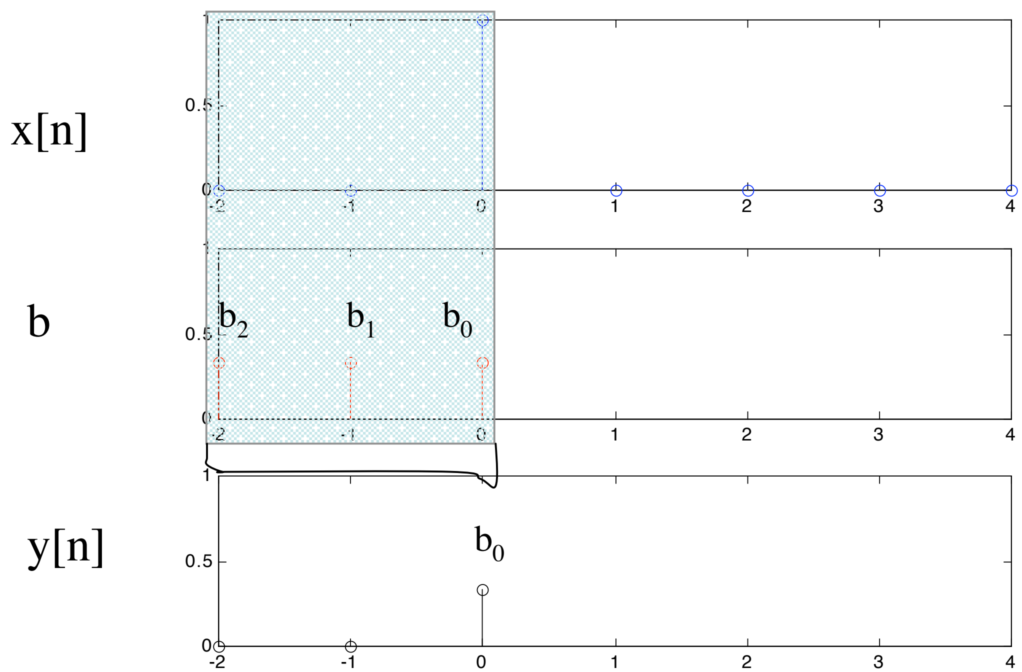
Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



$$y[0] = \frac{1}{3}\delta[0] + \frac{1}{3}\delta[-1] + \frac{1}{3}\delta[-2]$$

$$= \frac{1}{3}1 + \frac{1}{3}0 + \frac{1}{3}0 = \frac{1}{3}$$

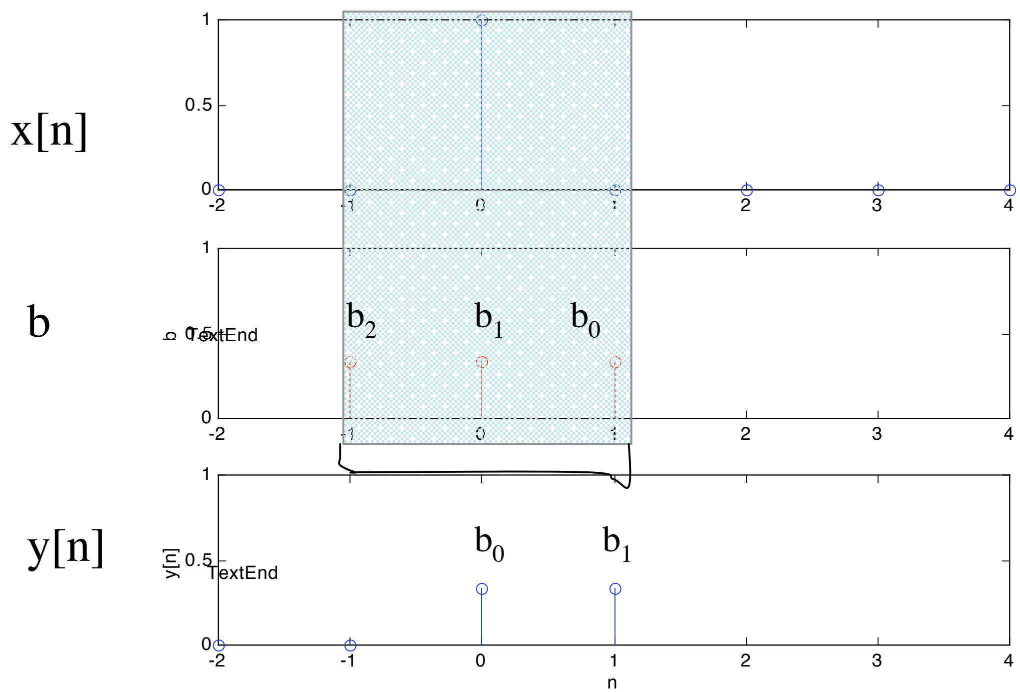
Impulse response of 3 pt. average

$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$$x[n]=\delta[n]=\begin{cases}1 & n=0\\ 0 & otherwise\end{cases}$$

Delta function

$$y[n]=\frac{1}{3}\delta[n]+\frac{1}{3}\delta[n-1]+\frac{1}{3}\delta[n-2]$$



$$y[1]=\frac{1}{3}\delta[1]+\frac{1}{3}\delta[0]+\frac{1}{3}\delta[-1]$$

$$=\frac{1}{3}0+\frac{1}{3}1+\frac{1}{3}0=\frac{1}{3}$$

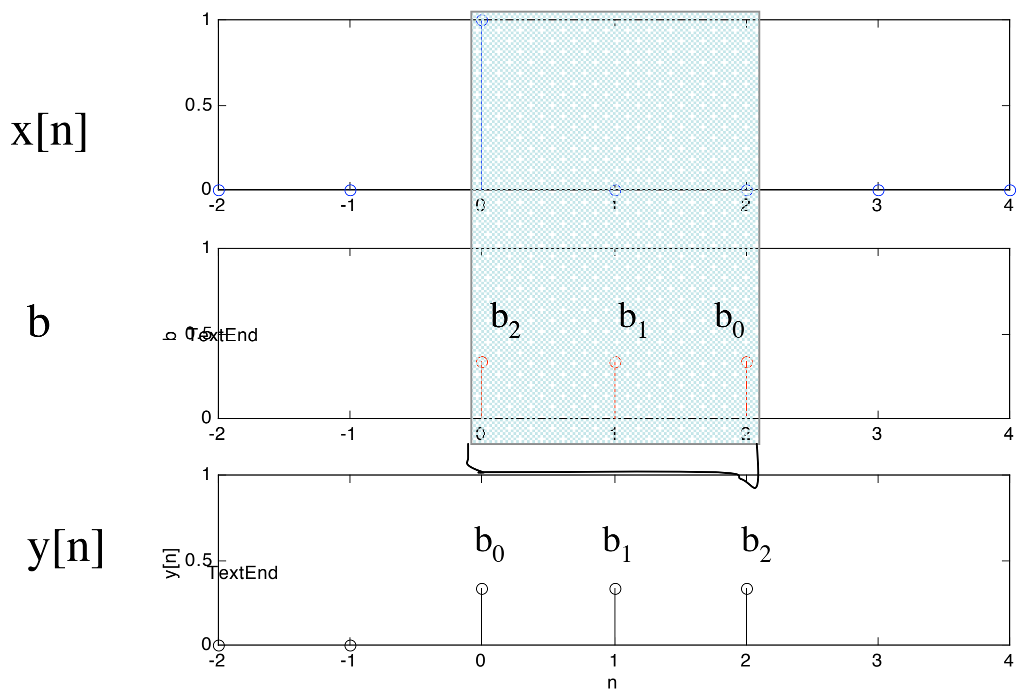
Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



$$y[2] = \frac{1}{3}\delta[2] + \frac{1}{3}\delta[1] + \frac{1}{3}\delta[0]$$

$$= \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}1 = \frac{1}{3}$$

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

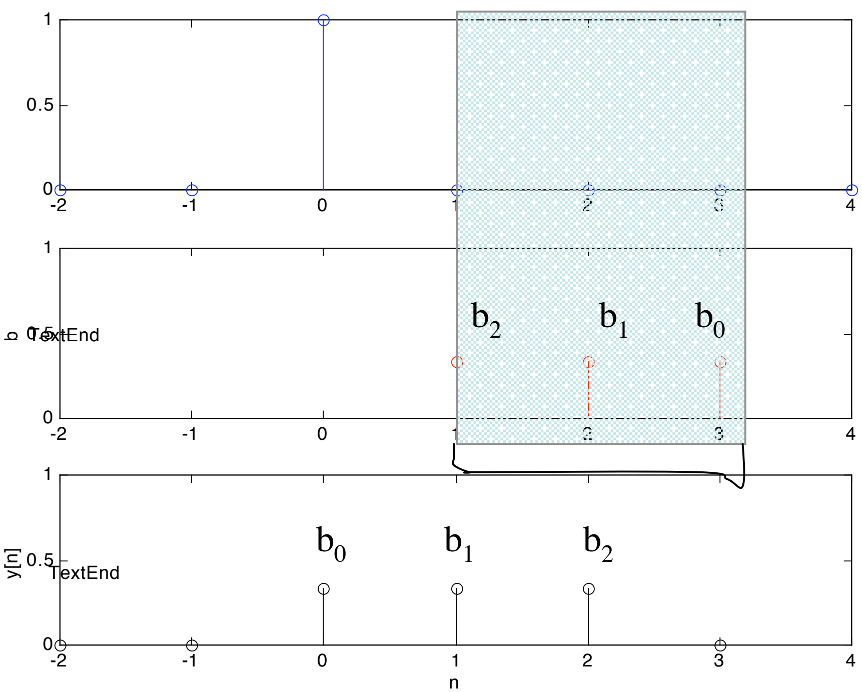
Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

x[n]

b

y[n]



$$y[3] = \frac{1}{3}\delta[3] + \frac{1}{3}\delta[2] + \frac{1}{3}\delta[1]$$

$$= \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}0 = 0$$

Impulse response of 3 pt. average

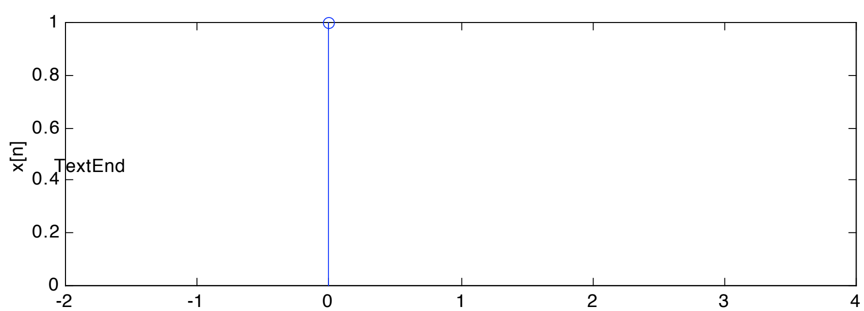
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

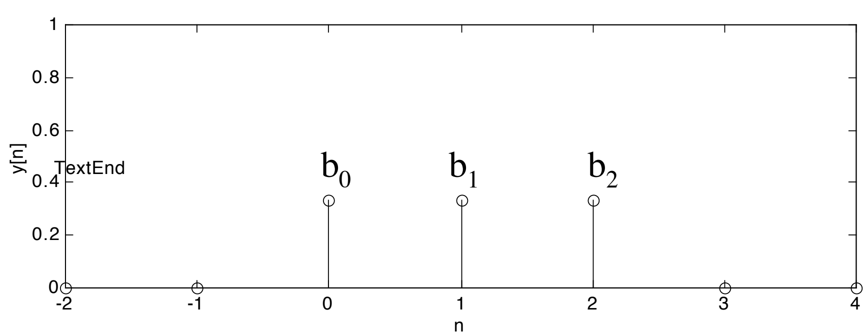
$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{0 \quad 0 \quad \tfrac{1}{3} \quad \tfrac{1}{3} \quad \tfrac{1}{3} \quad 0 \quad 0\}$$

$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$

x[n]



y[n]



Coefficients from impulse response

$$x[n]=\delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

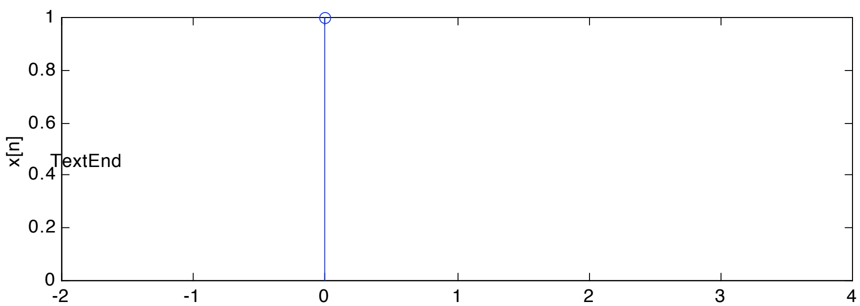
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

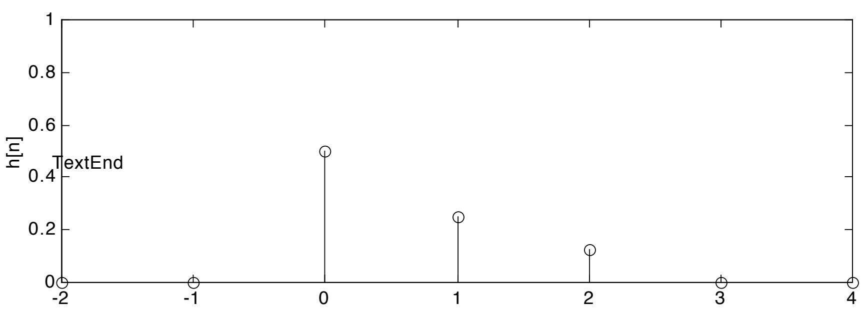
$$b_0, b_1, \ldots b_M = ???$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$$

x[n]



h[n]



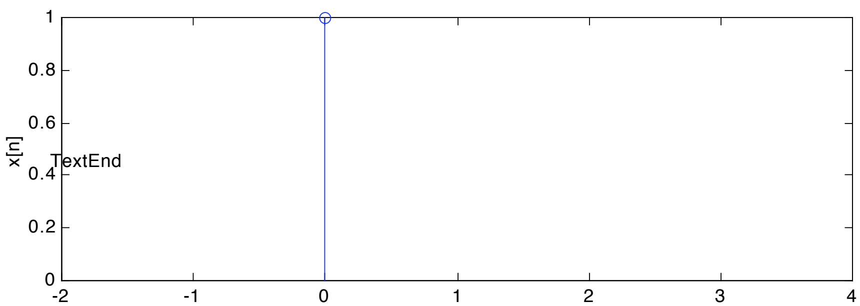
Coefficients from impulse response

$$x[n] = \delta[n]$$
$$= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

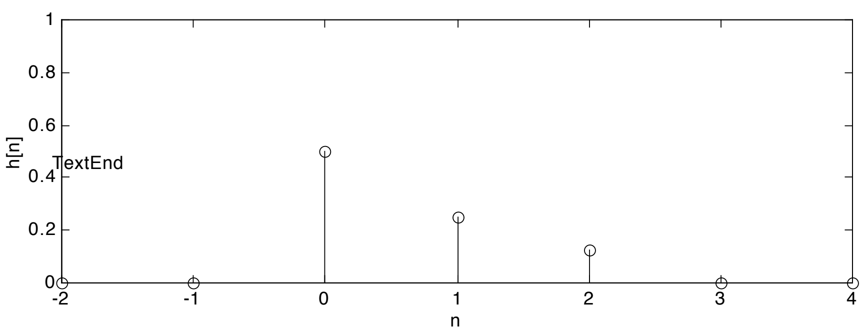
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
$$\{b_0, b_1, b_2\} = \left\{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\right\}$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]}$$
$$= \left\{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\right\}$$
$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$

x[n]



h[n]



Response from 2 impulses

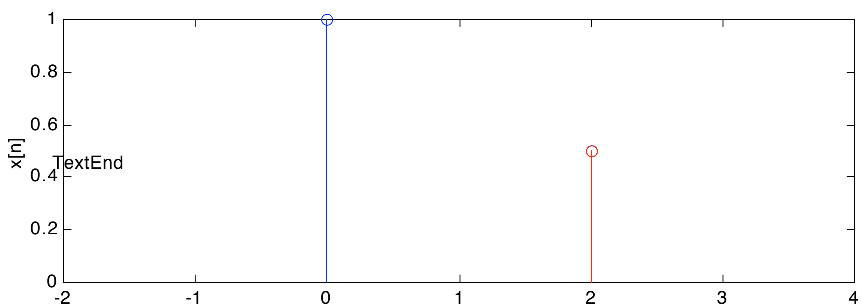
$$x[n] = \delta[n]$$
$$= \{0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0\}$$
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$\longrightarrow \boxed{\begin{array}{c} y[n] = \sum_{k=0}^{M} b_k x[n-k] \\ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \end{array}} \longrightarrow$$

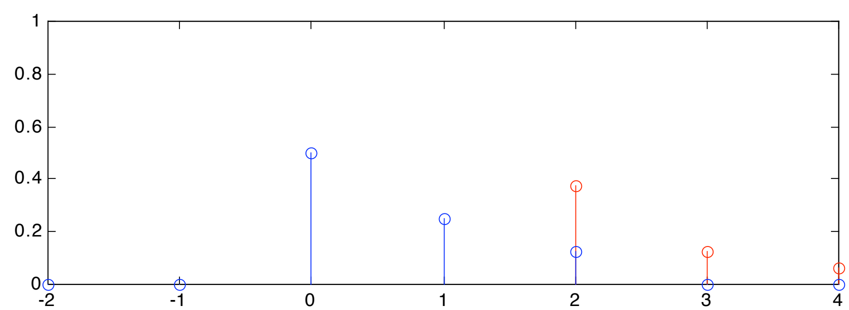
$$y[n] =$$
$$\{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} + \tfrac{4}{16} \quad \tfrac{2}{8} \quad \tfrac{1}{16}\}$$

$$y[n] = \{0 \quad 0 \quad h[0]x[0] \quad h[1]x[0] \quad h[2]x[0] + h[0]x[2] \quad h[1]x[2] \quad h[2]x[2]\}$$

x[n]



y[n]



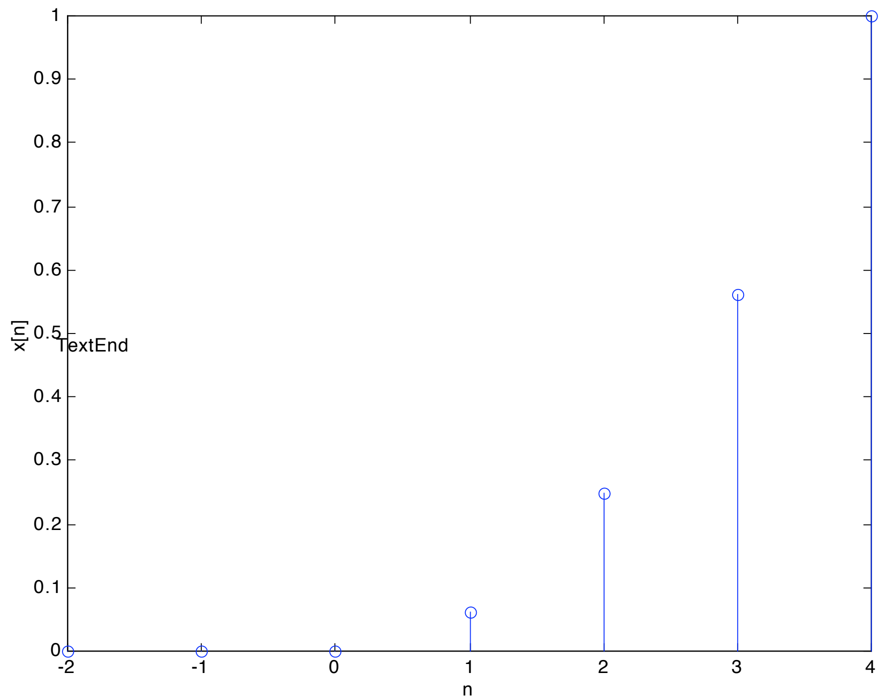
Sum the responses of each impulse

$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$x[n] = \left\{0 \quad 0 \quad 0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16}\right\}$$

$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$



$$x[n] = 0 \cdot \delta[n] + \tfrac{1}{16} \cdot \delta[n-1] + \tfrac{4}{16} \cdot \delta[n-2] + \tfrac{9}{16} \cdot \delta[n-3] + \tfrac{16}{16} \cdot \delta[n-4]$$

Any discrete signal be thought of a weighted sum of delayed impulses

Response from 2 impulses

$$x[n] = \delta[n]$$
$$= \{0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0\}$$
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

→

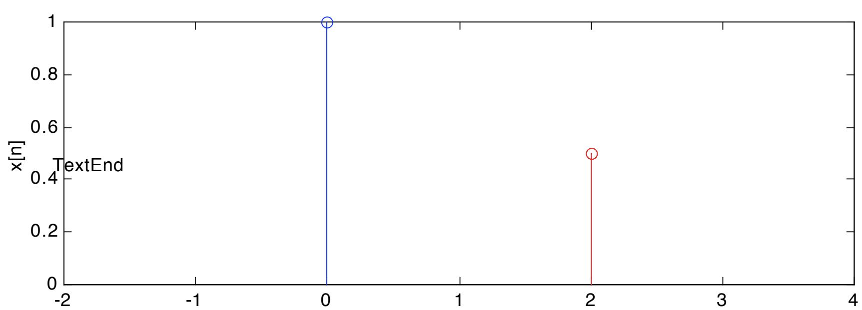
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
$$h[n] = \{b_0, b_1, b_2\} = \{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\}$$

→

$$y[n] =$$
$$\{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} + \tfrac{4}{16} \quad \tfrac{2}{8} \quad \tfrac{1}{16}\}$$

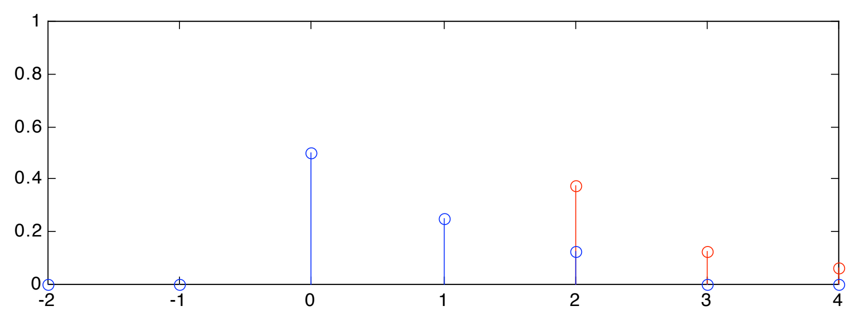
$$y[n] = \{0 \quad 0 \quad h[0]x[0] \quad h[1]x[0] \quad h[2]x[0] + h[0]x[2] \quad h[1]x[2] \quad h[2]x[2]\}$$

x[n]



$$y[2] = h[0]x[2-0] +$$
$$h[1]x[2-1] +$$
$$h[2]x[2-2]$$

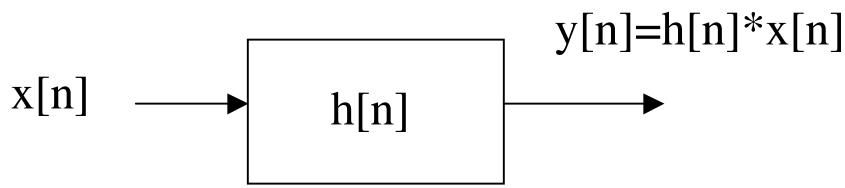
y[n]

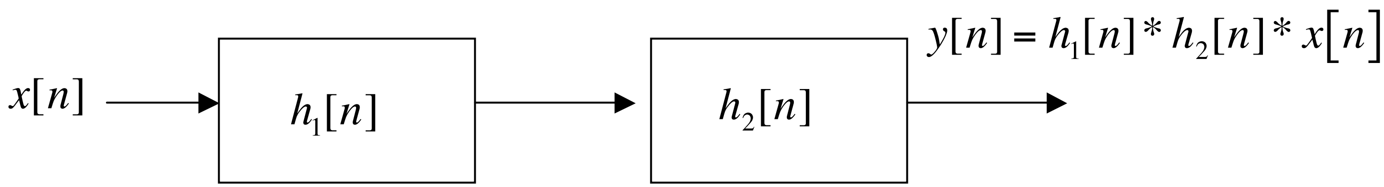


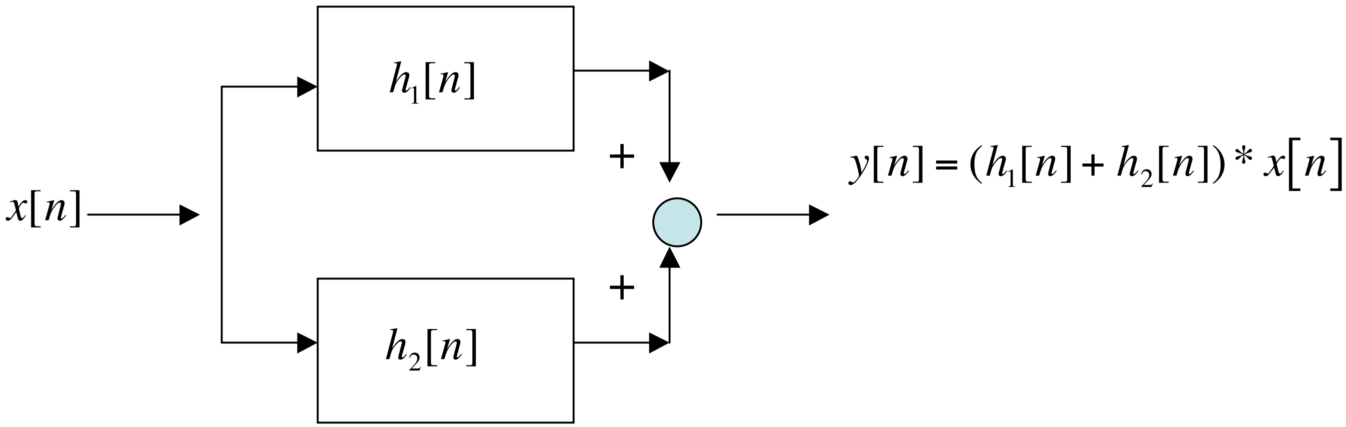
$$y[n] = \sum_{k=0}^{3} h[k]x[n-k]$$

Convolution sum

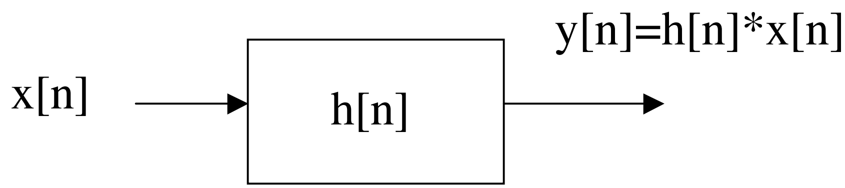
LTI Systems

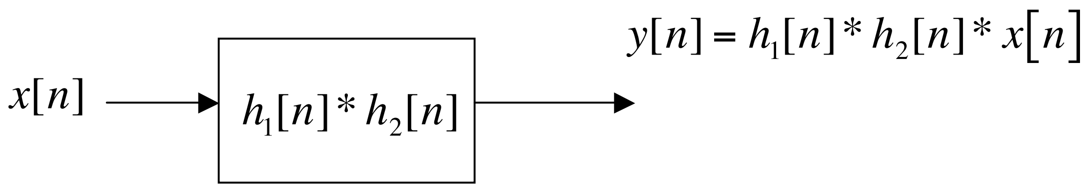


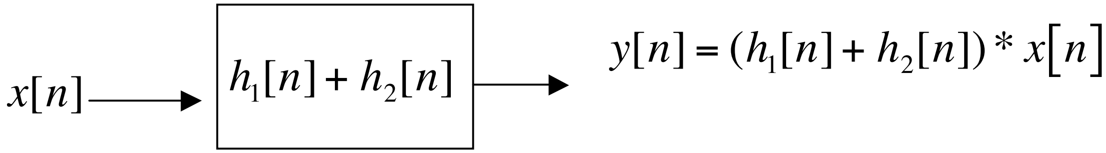




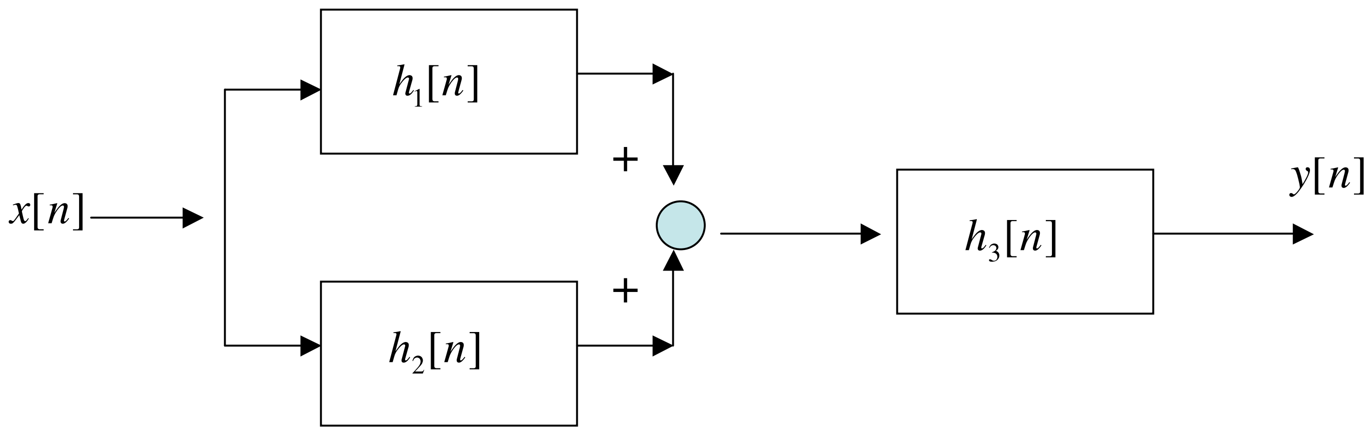
LTI Systems



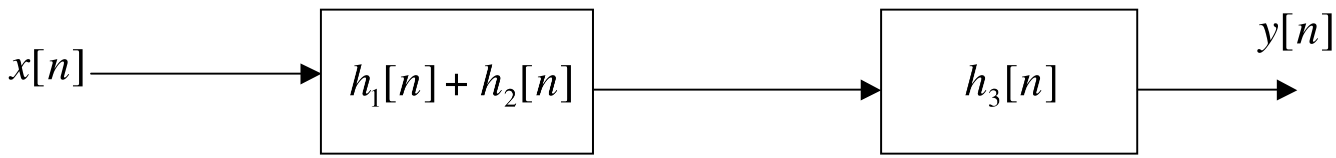




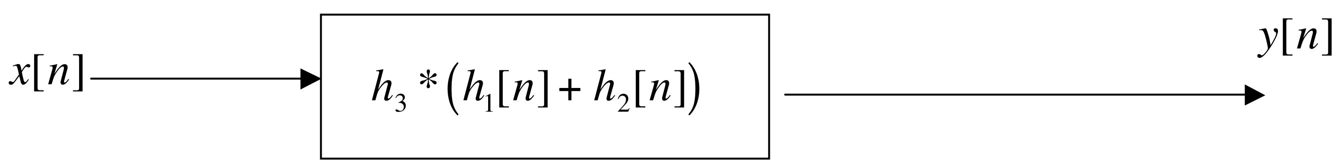
LTI Systems



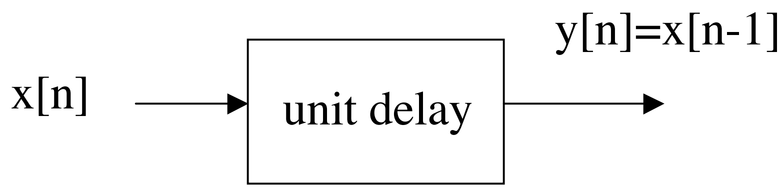
LTI Systems

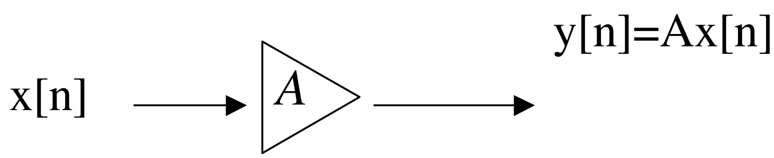


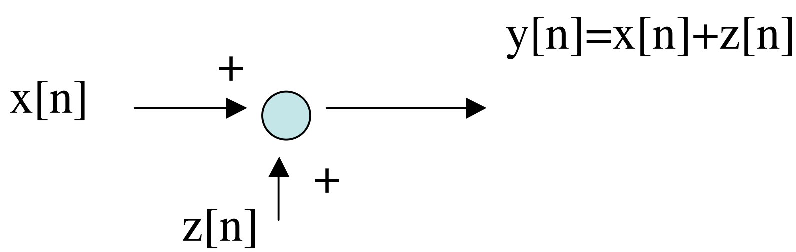
LTI Systems



Block Diagrams







Block Diagrams: Direct Form

$$x[n] = \delta[n]$$
$$= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$\longrightarrow$

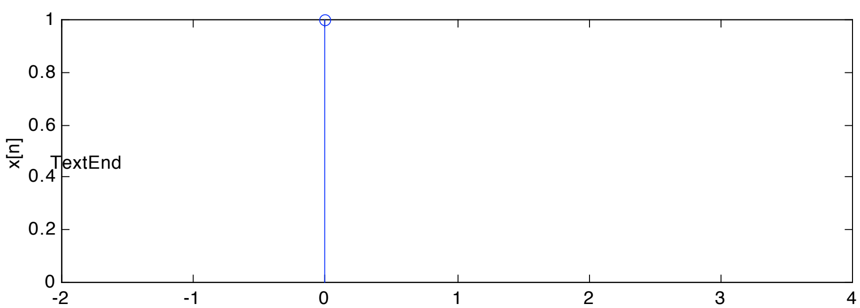
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
$$\{b_0, b_1, b_2\} = \{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\}$$

L=3, M=L-1=2

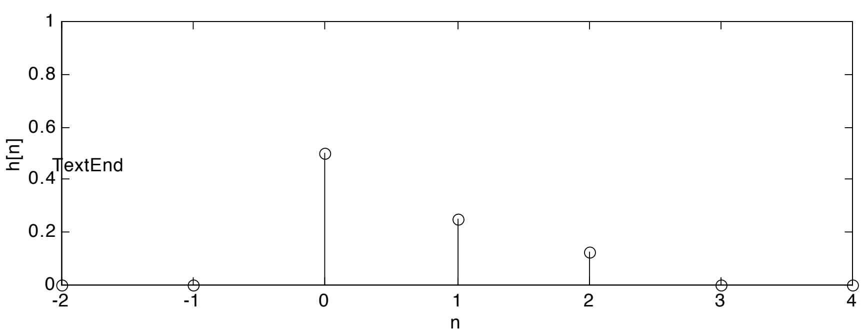
$\longrightarrow$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]}$$
$$= \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$$
$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$

x[n]



h[n]



Block Diagrams: Direct Form

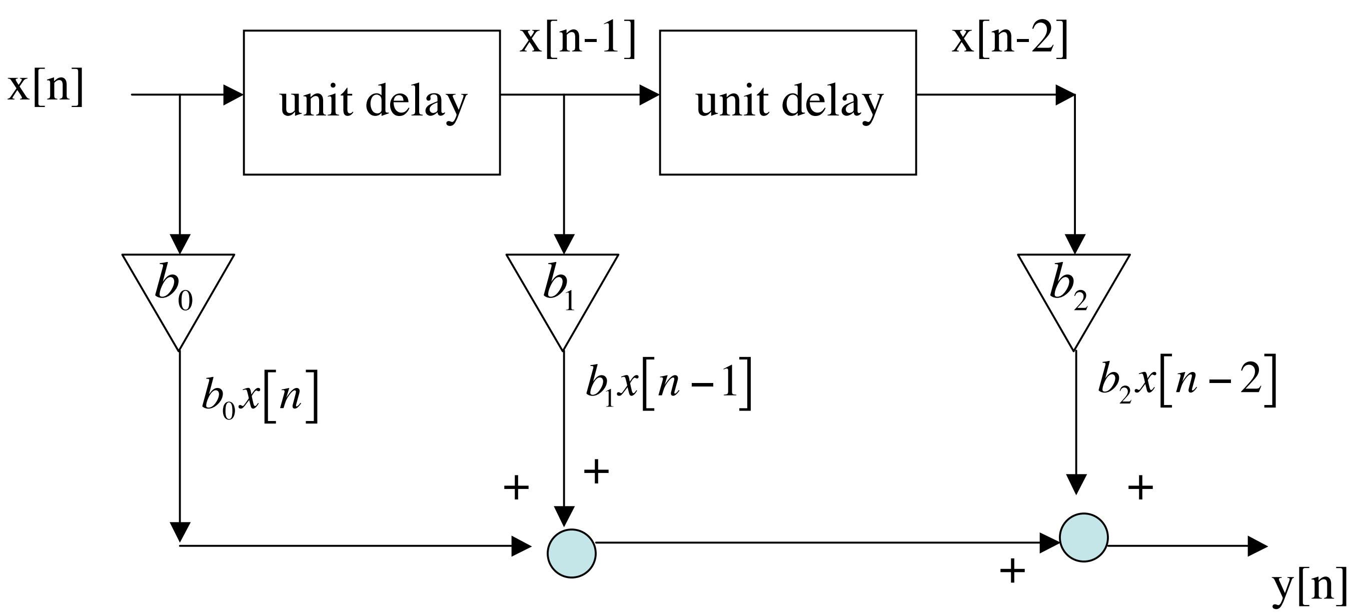
$$x[n] = \delta[n]$$
$$= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$
$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
$$\{b_0, b_1, b_2\} = \{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\}$$

L=3, M=L-1=2

$$h[n] = y[n]\big|_{x[n]=\delta[n]}$$
$$= \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$$
$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$



Block Diagrams: Transpose Form

$x[n] = \delta[n]$

$= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$

$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

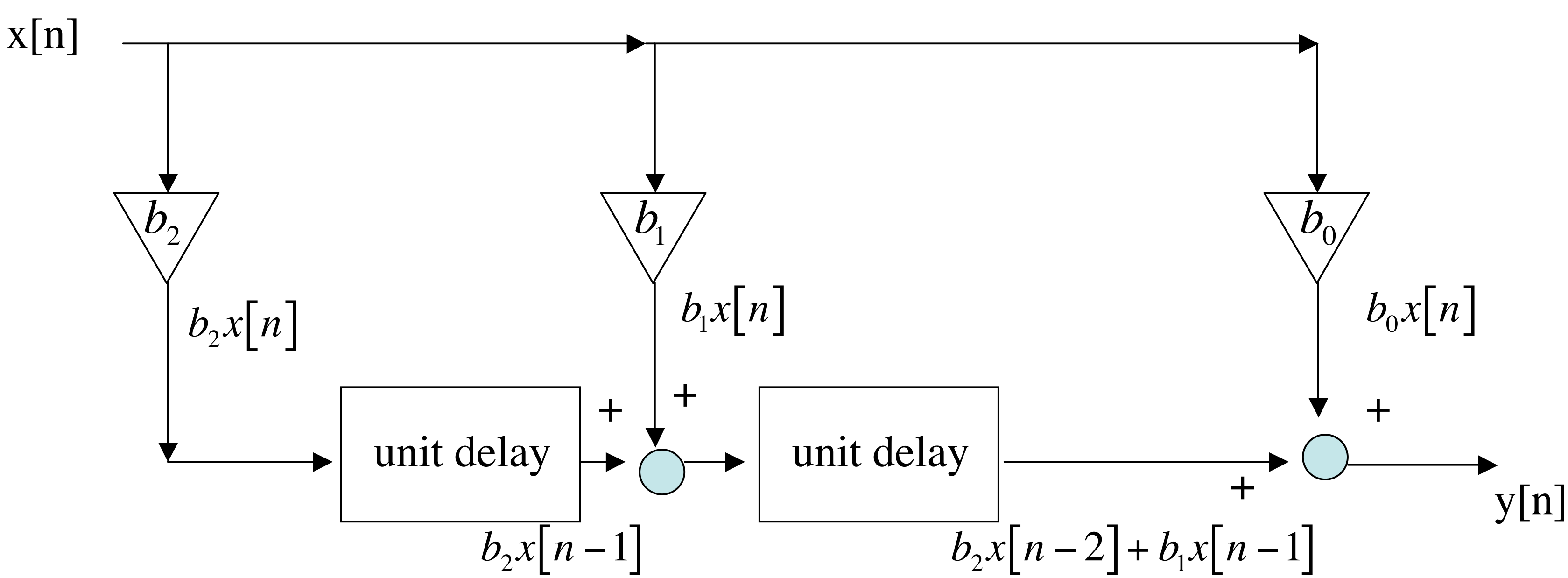
$$\{b_0, b_1, b_2\} = \{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\}$$

$h[n] = y[n]\big|_{x[n]=\delta[n]}$

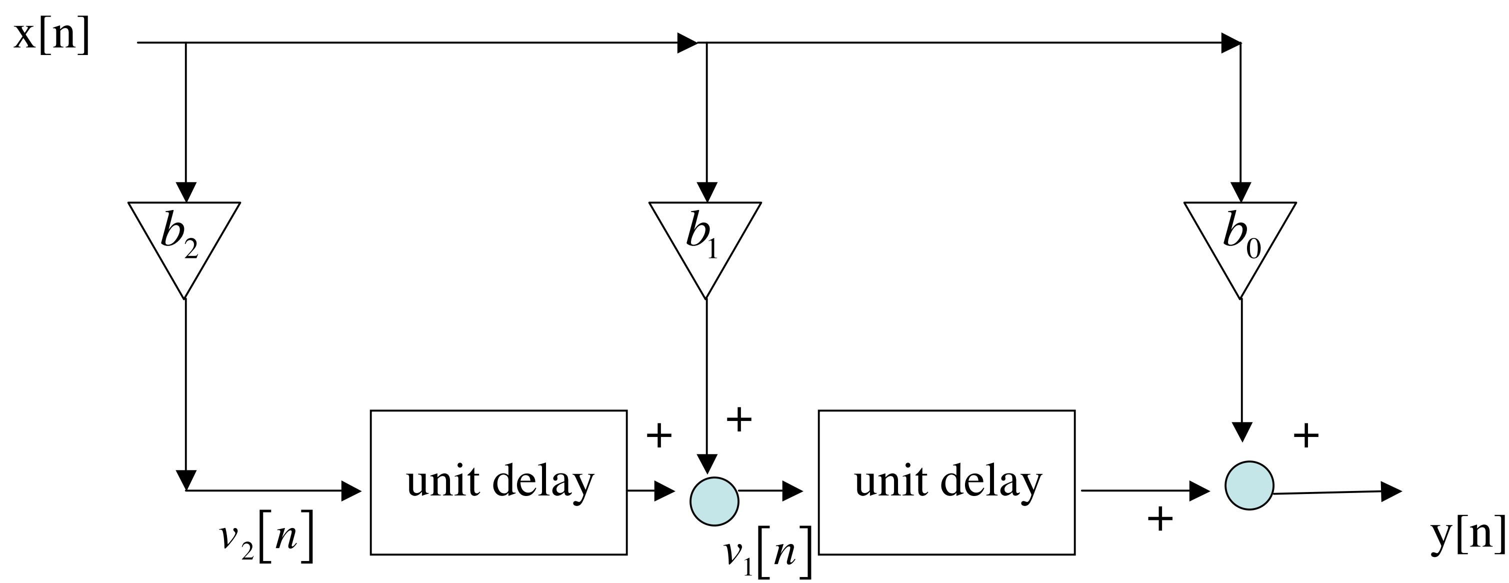
$= \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$

$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$



Block Diagrams to Difference Equations



$$v_2[n] = b_2 x[n]$$

$$v_1[n] = b_1 x[n] + v_2[n-1]$$

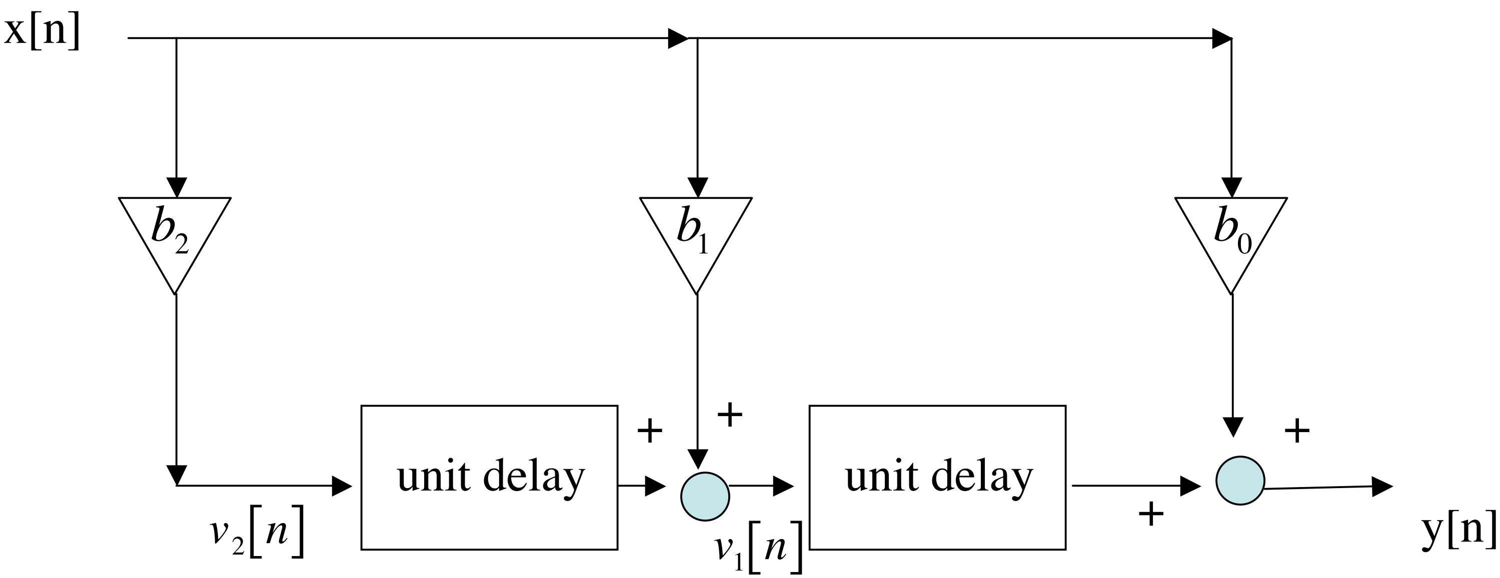
$$y[n] = b_0 x[n] + v_1[n-1]$$

$$y[n] = b_0 x[n] + v_1[n-1]$$

$$v_1[n] = b_1 x[n] + v_2[n-1]$$

$$v_2[n] = b_2 x[n]$$

Block Diagrams to Difference Equations



$v_2[n] = b_2 x[n]$

$v_1[n] = b_1 x[n] + v_2[n-1]$

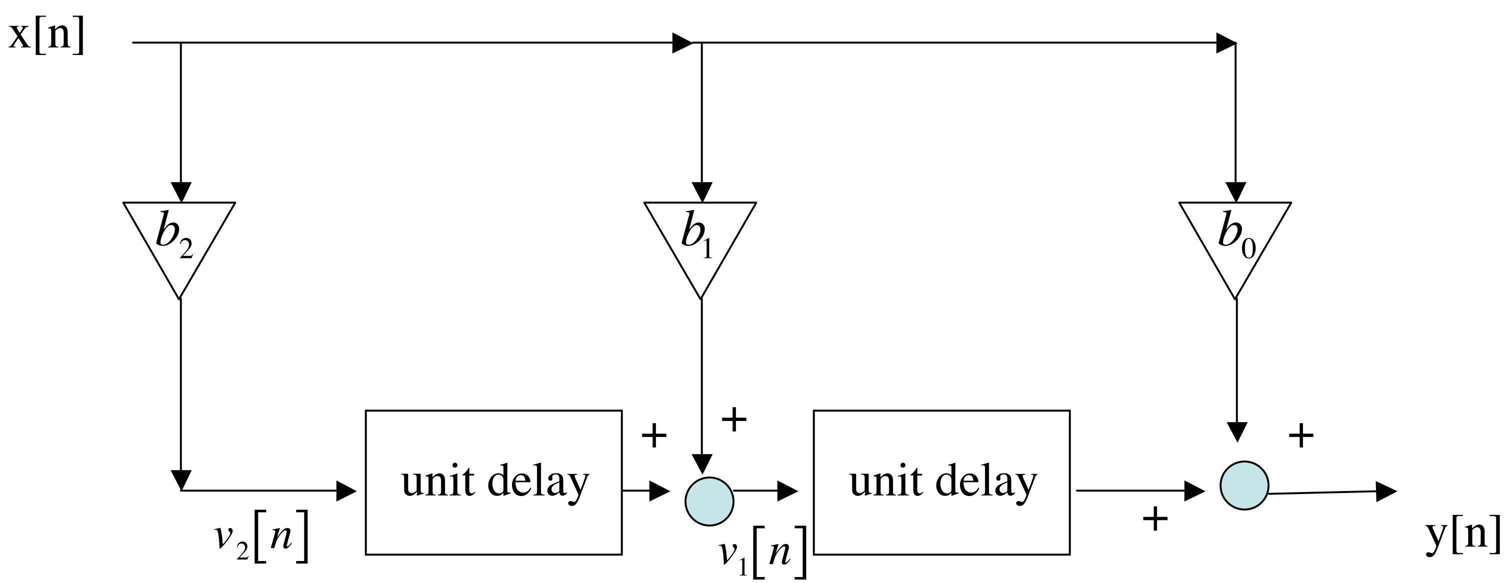
$y[n] = b_0 x[n] + v_1[n-1]$

$$y[n] = b_0 x[n] + v_1[n-1]$$

$$v_1[n-1] = b_1 x[n-1] + v_2[n-2]$$

$$v_2[n-2] = b_2 x[n-2]$$

Block Diagrams to Difference Equations

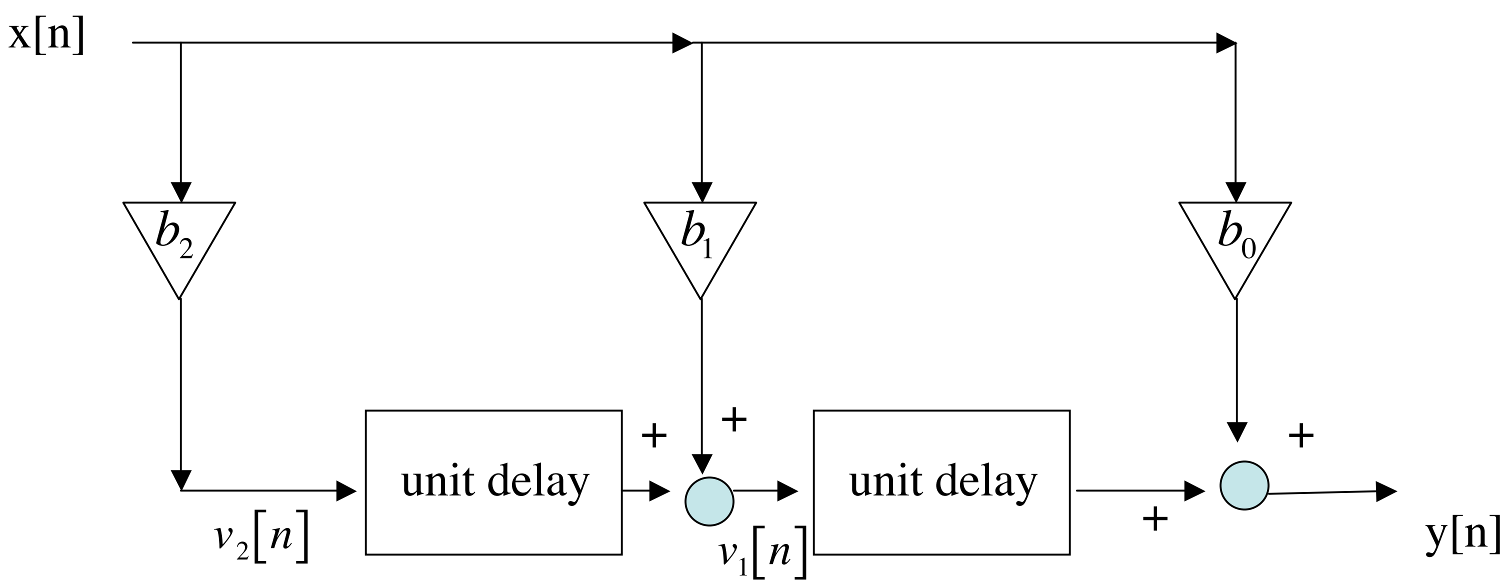


$v_2[n] = b_2 x[n]$ $\quad$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $\quad$ $y[n] = b_0 x[n] + v_1[n-1]$

$$y[n] = b_0 x[n] + v_1[n-1]$$

$$v_1[n-1] = b_1 x[n-1] + b_2 x[n-2]$$

Block Diagrams to Difference Equations



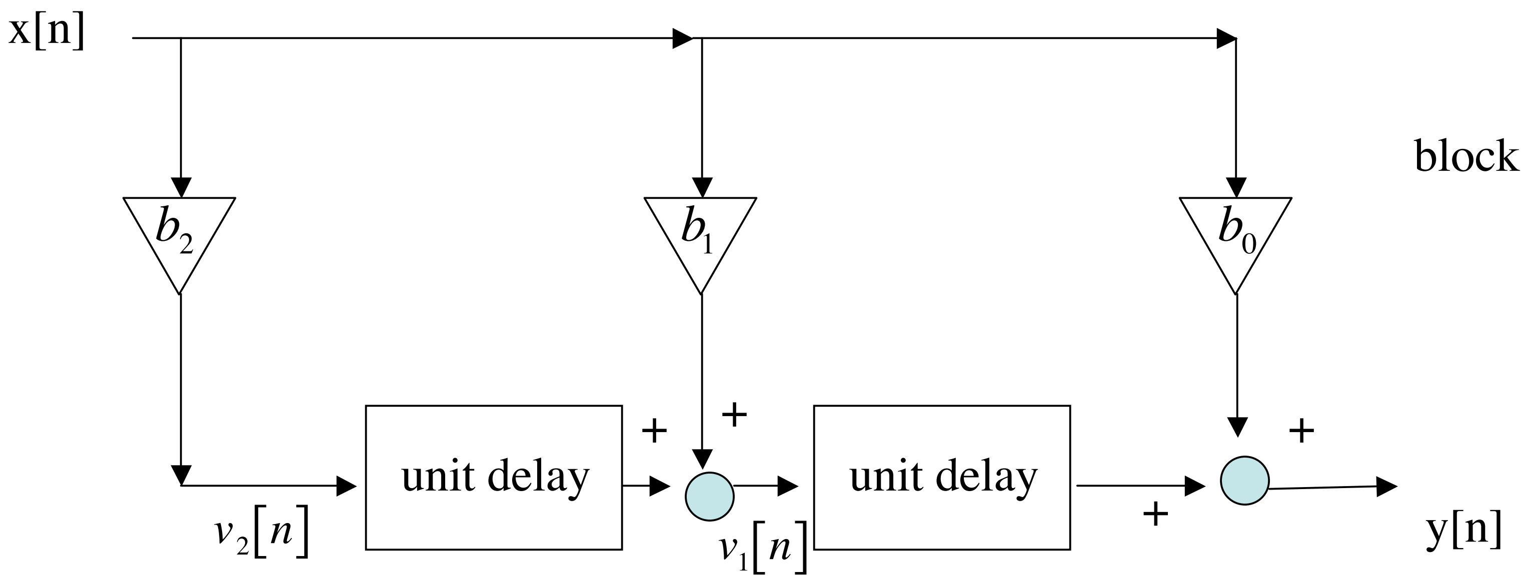
$v_2[n] = b_2 x[n]$

$v_1[n] = b_1 x[n] + v_2[n-1]$

$y[n] = b_0 x[n] + v_1[n-1]$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Block Diagrams to Difference Equations



block diagram

$$v_2[n] = b_2 x[n]$$

$$v_1[n] = b_1 x[n] + v_2[n-1]$$

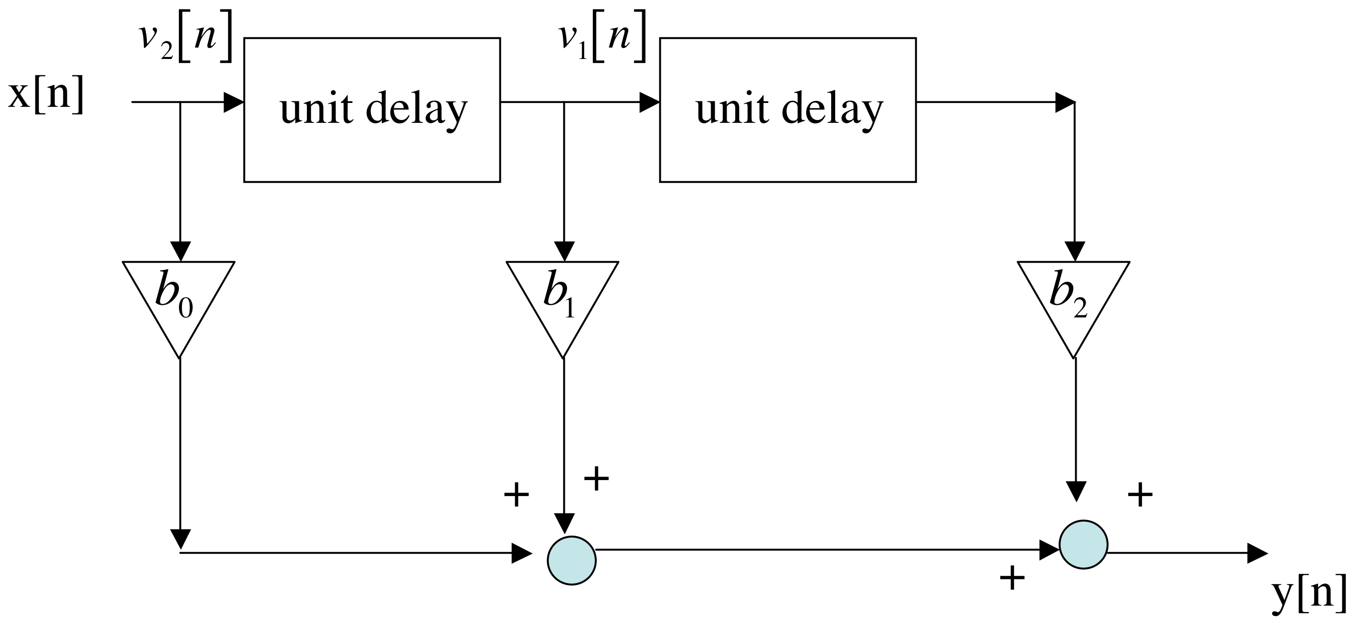
$$y[n] = b_0 x[n] + v_1[n-1]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$ difference equation

$$h[n] = b_0 \delta[n] + b_1 \delta[n-1] + b_2 \delta[n-2]$$ impulse response

equivalent ways
of describing system

Block Diagrams to Difference Equations



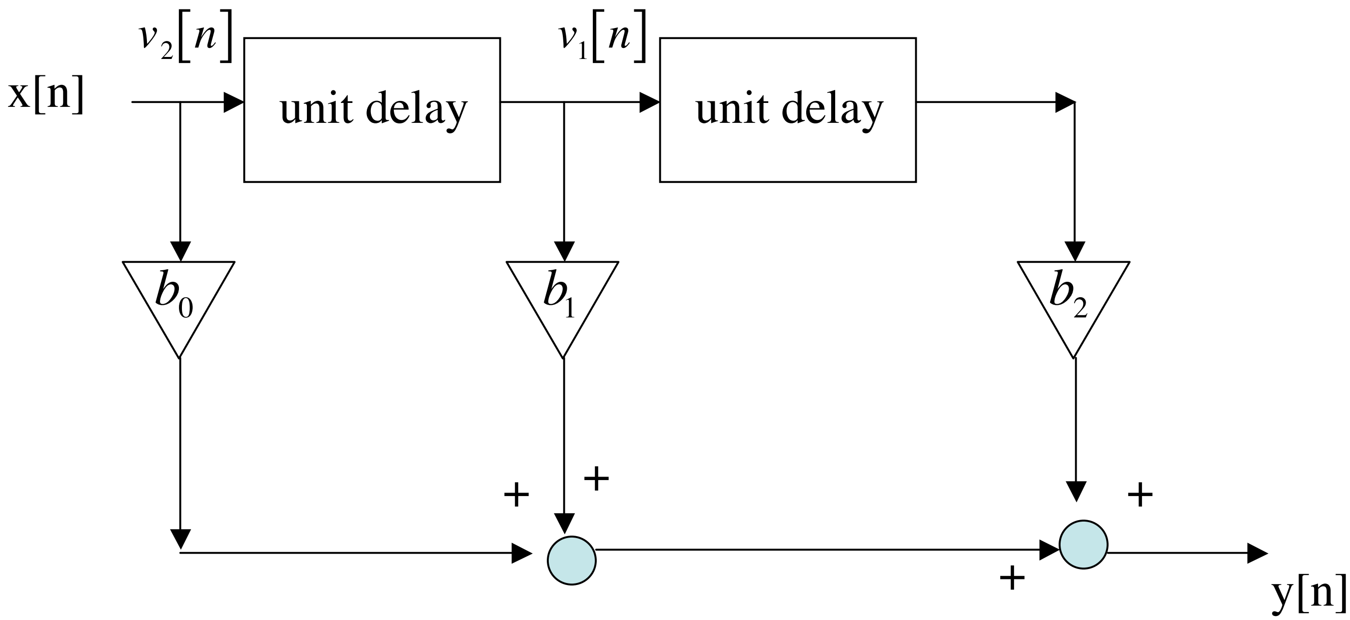
$v_2[n] = x[n]$

$v_1[n] = v_2[n-1]$

$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$

$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$
$$v_1[n] = v_2[n-1]$$
$$v_2[n] = x[n]$$

Block Diagrams to Difference Equations



$v_2[n] = x[n]$

$v_1[n] = v_2[n-1]$

$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$

$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$

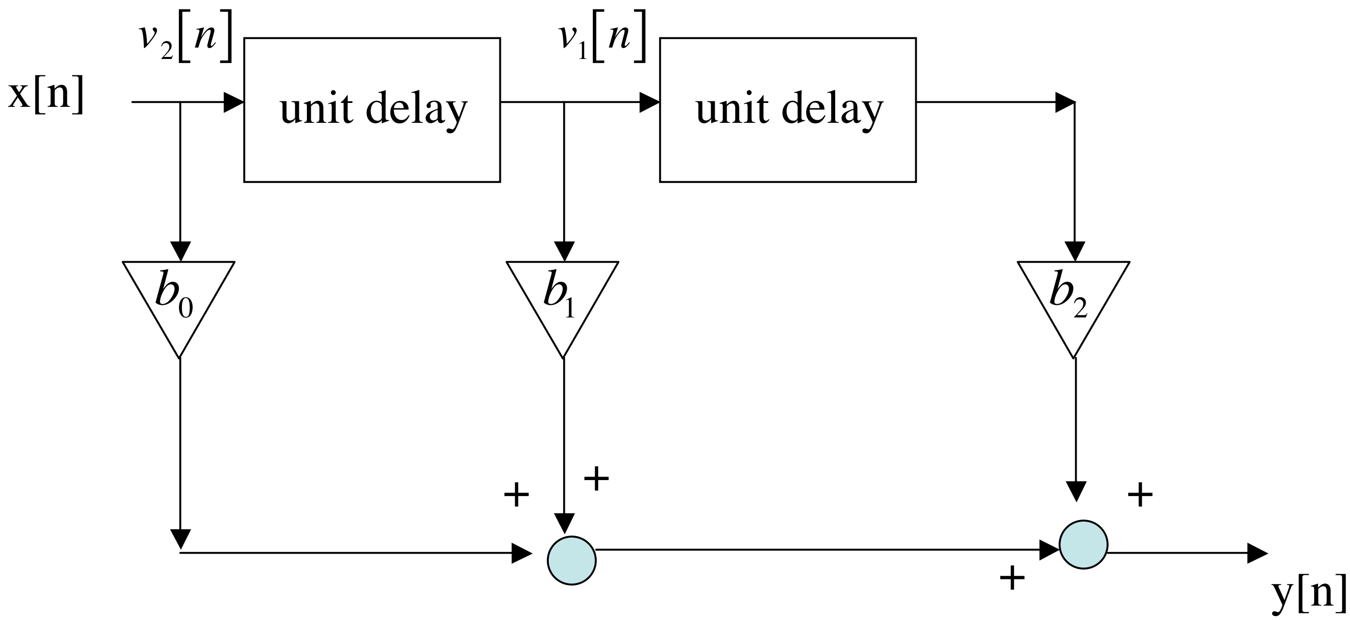
$$v_2[n] = x[n] \qquad v_1[n-1] = v_2[n-2]$$

$$v_2[n-2] = x[n-2]$$

$$v_1[n] = v_2[n-1]$$

$$v_2[n-1] = x[n-1]$$

Block Diagrams to Difference Equations



$$v_2[n] = x[n]$$

$$v_1[n] = v_2[n-1]$$

$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Homework:

p5_1: $$y(n) := \frac{1}{L}\cdot\left[\sum_{k=0}^{L-1} a^{n-k}\cdot u(n-k)\right]$$

L-point running average
for input sequence
$x[n]=a^n u[n]$, $n \geq 0$

hint: $$\sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$$

let z=n-k
k=n-z

$$\frac{1}{L}\cdot\left[\sum_{n-z=0}^{L-1} a^z\cdot u(z)\right]$$

$$\frac{1}{L}\cdot\left[\sum_{z=n}^{n-(L-1)} a^z\cdot u(z)\right]$$

remember $n \geq 0$

p5_6: FIR & delays

Homework:

L-point running average
for input sequence
$x[n]=a^n u[n],\ n \geq 0$

p5_1: $y(n) := \frac{1}{L} \cdot \left[\sum_{k=0}^{L-1} a^{n-k} \cdot u(n-k) \right]$

hint: $\sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$

let z=n-k
k=n-z

$\frac{1}{L} \cdot \left[\sum_{n-z=0}^{L-1} a^z \cdot u(z) \right]$

$\frac{1}{L} \cdot \left[\sum_{z=n}^{n-(L-1)} a^z \cdot u(z) \right]$

remember $n \geq 0$

p5_6: FIR & delays
FIR and single delay

$y[n]=ax[n]+bx[n-1]$

3 point average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2] \qquad x[n] = \sin(2\pi n/15)\cdot u[n] \qquad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$