Student Name: _____

MAS 622J/1.126J: Pattern Recognition and Analysis

Midterm Quiz, October 22, 2008

This quiz consists of 3 problems, 8 pages. Please make sure you have all 8 pages.

1. (40 points) Is the statement True or False? Even if you get the right answer, you must explain your reasoning to get full credit. Feel free to make a sketch, show mathematically, or just explain in English, giving justification or a counterexample.

   For all these questions, assume a two-class classification problem, where each class has a Gaussian distribution:

   (a) Assume zero-one costs. The minimum probability of error decision boundary always passes through the point where the two classes' likelihood functions are equal.

   (b) Assume d=2 dimensions, and that the two classes have the same non-diagonal covariance matrix. The decision boundary that minimizes the probability of error is always a straight line.

(c) Assume d=2 dimensions, and the two classes have covariance matrices equal to the Identity matrix. The decision boundary that minimizes the probability of error will always pass in between the means of the two classes. (It will always cross the line connecting their means.)

(d) Assume d=50 dimensions, and the two classes have covariance matrices that are equal to the identity matrix. The optimal Fisher linear projection for separating the classes is to project onto a line connecting the means of the two class densities.

This is an extra page to give you space to show all your work for partial credit.

2. (30 points) Sarah has a big speech to make and wants to get good ratings. She knows that how well she sleeps and what kind of advice she gets will influence her ability to prepare well. She thinks the ratings of her speech will be influenced not only by how well she prepares, but also by the weather (ask Roz if you want to learn about research showing evidence of the latter.) Sarah makes a belief net to analyze her chances of getting great ratings. Here are the variables:

   a_1 = sleeps well, a_2 = sleeps poorly

   b_1 = gets good advice, b_2 = gets poor advice

   c_1 = prepares well, c_2 = prepares poorly

   d_1 = good weather, d_2 = poor weather

   e_1 = gets good ratings, e_2 = gets poor ratings.

   You can work these parts independently

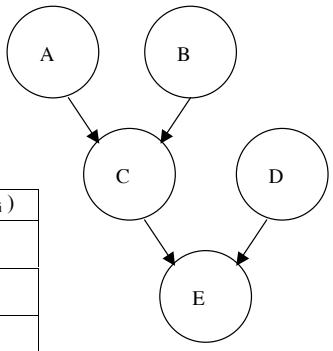
   (a) What is the probability that Sarah gets good ratings given everything goes well (she sleeps well, gets good advice, prepares well, and the weather is good)?

   (b) What is the probability that Sarah gets good ratings given she prepares well, but the weather is bad? Hint: You can answer this without doing any math.

   (c) Suppose Sarah gets a poor rating despite having prepared well and the weather being good; what is the probability that this was due to sleeping poorly, i.e. find $p(a_2|c_1, d_1, e_2)$.

$P(a_1)$	$P(a_2)$
0.9	0.1

$P(b_1)$	$P(b_2)$
0.9	0.1



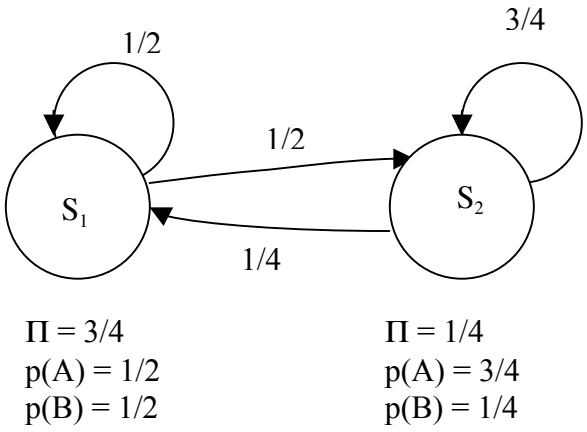
$P(d_1)$	$P(d_2)$
0.8	0.2

	$P(c_1 \mid a_i, b_i)$	$P(c_2 \mid a_i, b_i)$
a_1, b_1	0.9	0.1
a_1, b_2	0.7	0.3
a_2, b_1	0.8	0.2
a_2, b_2	0.6	0.4

	$P(e_1 \mid c_i, d_i)$	$P(e_2 \mid c_i, d_i)$
c_1, d_1	0.8	0.2
c_1, d_2	0.7	0.3
c_2, d_1	0.6	0.4
c_2, d_2	0.5	0.5

People often miss points by forgetting to check if they worked every part of the problem. Please check that you answered all parts of every problem, at least with your best partial guess if you don't get to construct a full answer.

3. (30 points) Suppose you have a two-state HMM defined by the parameters given below.



(a) Which is a more likely observation sequence for this HMM: sequence1: $\mathbf{A}, \mathbf{A}, \mathbf{A}$ or sequence2: $\mathbf{A}, \mathbf{A}, \mathbf{B}$. Find the likelihood for each sequence of observations.

(b) (Independent of the part above): What is the most likely sequence of states that gives the sequence $\mathbf{A}, \mathbf{A}, \mathbf{A}$.

Please show all your work for partial credit; please check that you answered all parts of each problem.