MAS 160/510 Problem Set 1 Solutions

1. **Perdiodicity**

   Determine the fundamental period of each of the following signals;

   (a) $x_a(t) = \cos(5\pi t)$

   (b) $x_a(t) = 8\cos(4t + \frac{\pi}{12})$

   (c) $x[n] = \cos(0.03\pi n)$

   (d) $x[n] = \cos(5\pi n) + \cos(\frac{4}{5}\pi n)$

   Solutions:

   (a) $T = \frac{2\pi}{5\pi} = \frac{2}{5}$

   (b) $T = \frac{2\pi}{4} = \frac{\pi}{2}$

   (c) $T_c = \frac{2\pi}{0.03\pi} = \frac{200}{3}$.
   However, since this function is discrete in time, the fundamental period should be the smallest integer for which function values begin to repeat. Thus, we have:
   $T = 3T_c = 200$

   (d) The fundamental period of $\cos(5\pi n)$ is $T_1 = \frac{2\pi}{5\pi} = \frac{2}{5}$. The fundamental period of $\cos(\frac{4}{5}\pi n)$ is $T_2 = \frac{2\pi}{\frac{4}{5}\pi} = \frac{5}{2}$. The fundamental period of a combination of the two signals is the least common multiple of the fundamental periods of each, which is $T = 10$.

2. **Sampling**

   (a) Let T_s, the sampling period for a continuous time signal, be $3/2$ and let $x(t) = \cos(2\pi t)$. Sketch $x[n] = x(nT)$ for $n = 0, 1, ..., 8$.

   (b) Let $T_s = 1/3$ and let $x(t) = \cos(2\pi t)$. Sketch $x[n] = x(nT)$ for $n = 0, 1, ..., 8$.
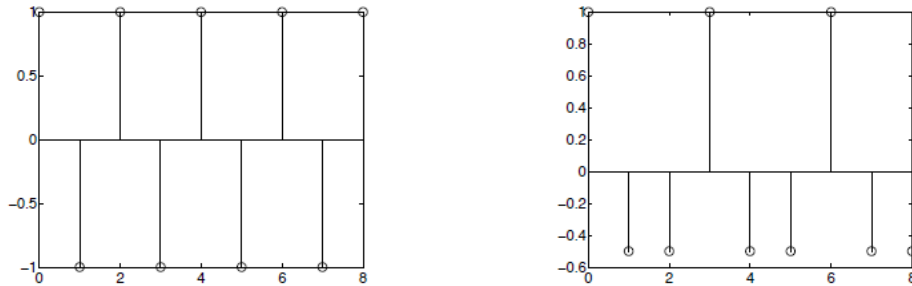

Figure 1: (a) $x[2n] = \cos(3\pi n)$ and (b) $x[n] = \cos(\frac{2}{3}\pi n)$

3. Review of Complex Numbers (*DSP First* problems at the end of Appendix A)

(a) A.3(b)

(b) A.4(a)-(e)

(c) A.5(e), (f), (i)

(d) A.8

Solutions:

(a) $e^{j(\pi+2\pi m)}$ (m an integer)
$e^{j(\pi+2\pi m)} = \cos(\pi + 2\pi m) + j\sin(\pi + 2\pi m) = -1$

(b) (a) $3e^{j2\pi/3} - 4e^{-j\pi/6}$
$= 3(\cos(2\pi/3 + j\sin(2\pi/3)) - 4(\cos(-\pi/6) + j\sin(-\pi/6))$
$= 3(-1/2 + j\sqrt{3}/2) - 4(\sqrt{3}/2 - j/2)$
$= -(2\sqrt{3} + 3/2) + j(3\sqrt{3}/2 + 2) \approx -4.9641 + j4.5981 \approx 6.76643e^{j2.39445}$

(b) $(\sqrt{2} - j2)^8$
$= (\sqrt{6}e^{j\tan^{-1}(\sqrt{2})})^8$
$= 6^4 e^{j8\tan^{-1}(\sqrt{2})}$

If you just directly expand out the original expression you can get the answer without approximation $= 272 - j896\sqrt{2} \approx 272 - j1267.1$

(c) $(\sqrt{2} - j2)^{-1}$
$= \frac{1}{\sqrt{2}-j2} = \frac{\sqrt{2}-j2}{\sqrt{2}+j2} = \frac{\sqrt{2}+j2}{6} \approx 0.2357 + j0.3333 \approx 0.408248e^{j0.955317}$

(d) $(\sqrt{2} - j2)^{\frac{1}{2}}$
$= (\sqrt{6}e^{j\tan^{-1}(\sqrt{2})})^{\frac{1}{2}} = \sqrt[4]{6}e^{\frac{j}{2}\tan^{-1}(\sqrt{2})} \approx 1.3899 - j0.7195 \approx 1.56508e^{-j0.477658}$

(e) $\Im m\{je^{-j\pi/3}\}$
$= \Im m\{j[\cos(-\pi/3) + j\sin(-\pi/3)]\} = \Im m\{j[\cos(\pi/3) - j\sin(\pi/3)]\}$
$= \Im m\{j\cos(\pi/3) + \sin(\pi/3)]\} = \cos(\pi/3) = 1/2$

(c) $z_1 = -4 + j3 \approx 5e^{j2.4981}$
$z_2 = 1 - j \approx \sqrt{2}e^{-j0.7854}$

(e) $z_1^{-1} = 1/z_1$
$= 1/(-4 + j3) = \frac{-4-j3}{-4-j3} \cdot \frac{1}{-4+j3} = \frac{-4-j3}{16+12j-12j+9} = -\frac{4}{25} - j\frac{3}{25}$
$\approx 0.2e^{-j2.4981}$

(f) z_1/z_2
$= \frac{-4+j3}{1-j} = \frac{1+j}{1+j} \cdot \frac{-4+j3}{1-j} = \frac{-4-4j+3j-3}{1+j-j+1} = -7/2 - j/2$
$\approx 5e^{j2.4981}/\sqrt{2}e^{-j0.7854} = 3.5355e^{j3.2835}$

(i) $z_1 z_2$
$= (-4 + j3)(1 - j) = -4 + 4j + 3j + 3 = -1 + 7j$
$\approx 5e^{j2.4981}\sqrt{2}e^{-j0.7854} = 7.071e^{j1.7127}$

(d) $z^4 = j$
We expect to see 4 distinct solutions. We first note that we can rewrite in the right hand side of the equation in exponential form $j = 1^4 \times e^{j\pi/2+j2n\pi}$ for integer n. Thus we have:
$z^4 = 1^4 \times e^{j\pi/2+j2n\pi} \Rightarrow z_n = 1 \times e^{j\frac{\pi}{8}+j\frac{2n\pi}{4}}$
$\Rightarrow z_0 = e^{j\pi/8}, z_1 = e^{j5\pi/8}, z_2 = e^{j9\pi/8}, z_3 = e^{j13\pi/8}$ for values $n = 0, 1, 2, 3$
if we try $n = 4$, we see that $z_4 = e^{j17\pi/8} = e^{j\pi/8+j2\pi} = e^{j\pi/8} = z_0$, so the roots start repeating.

4. **Matlab: Introduction** Once you have Matlab installed, these steps will guide you through the Matlab exercises (Note the "%" symbol indicates a comment and the text following needs not be entered).

(a) In the Matlab Command Window, type:

```
x = [1:1:150]; % sets up the domain
yCarrier = cos(pi*x/5); % create a sinusoid to act as a carrier
plot(x,yCarrier); % and plot it
ySignal = rand(size(yCarrier)); % create a signal to be transmitted
plot(x,ySignal); % and plot that too
ySum = yCarrier + ySignal; % create an average signal
yDiff = yCarrier - ySignal; % create a difference signal
```

(b) Using only linear combinations of the `ySum` and `yDiff` signals, recover the `ySignal` and plot it.
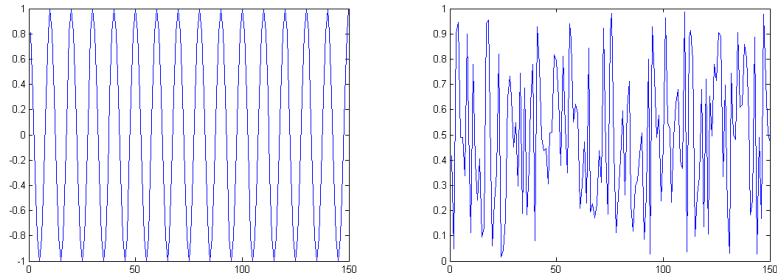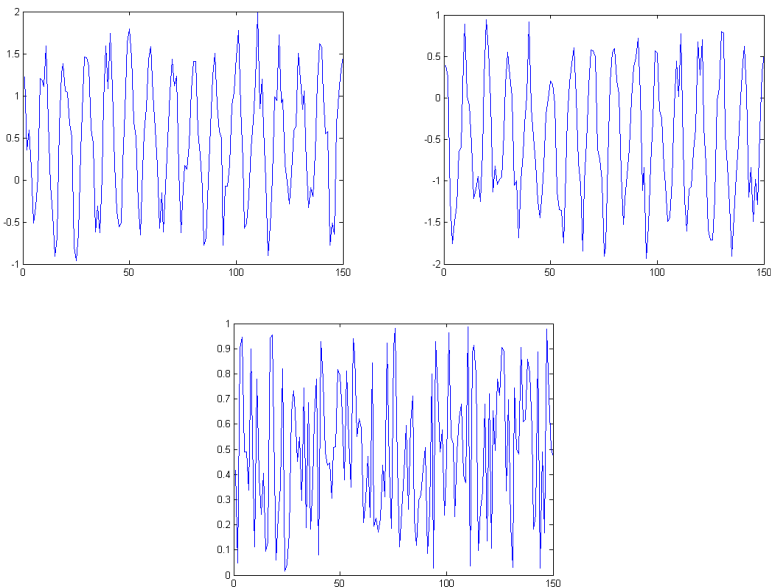
Solutions:

Figure 2: yCarrier and ySignal

Figure 3: ySum, yDiff, and yRecovered = 1/2(ySum−yDiff)

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The following MATLAB exercises (found in Appendix C of the *DSP First* text) should be treated as walkthrough tutorials to important concepts. You can ignore any references to *instructor verification* or a *lab report*. We will specify which items we would like turned in as part of the homework. However, it would be diffcult to do only the parts that are to be turned in (i.e. it would be unwise to skip steps in the lab).

5. ***DSP FIRST* Lab 1**

   Items to be turned in:

   (a) The `expand` function from C.1.2.6.

   (b) The `replacez` function from C.1.2.7.

   (c) Plots specified in C.1.3.1.

   Solutions:

   (a) ```
function Z = expand(xx, ncol)
    Z = repmat(x(:), 1, ncol);
```

   (b) ```
function Z = replacez(A)
    Z = A.*(A>0)+77*(A<0);
```
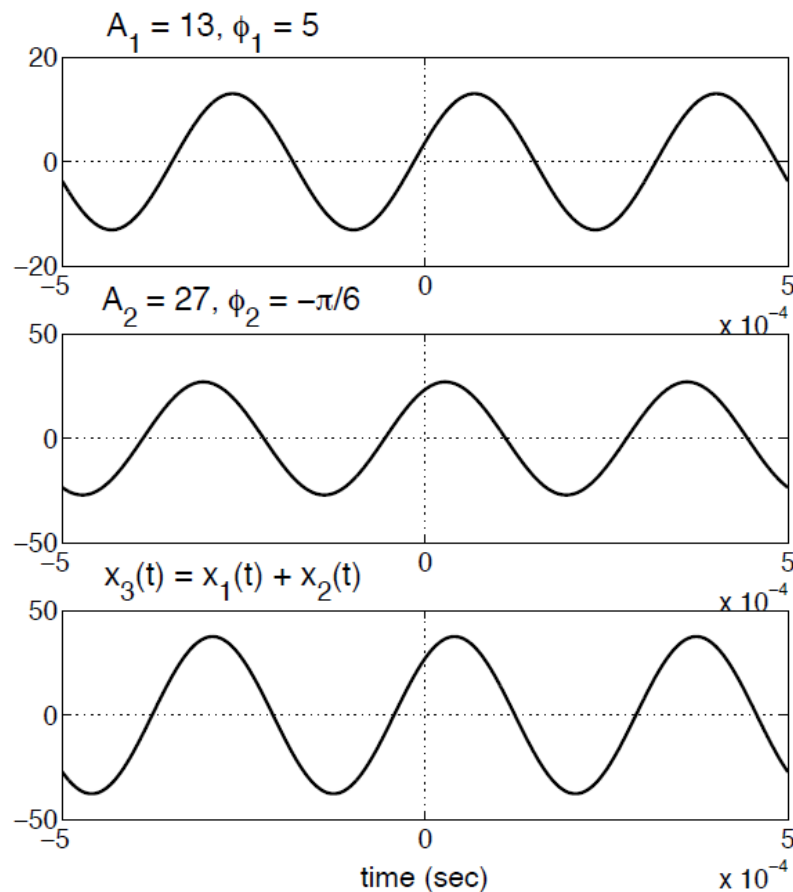
   (c)

Figure 4: Manipulating Sinusoids with MatLab - C.1.3.1

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6. ***DSP FIRST* Lab 2**

Items to be turned in:

(a) The `sumcos` function from C.2.2.2.

(b) Plots specified in C.2.3.2.

(c) Plots and answers to questions specified in C.2.4.

Solutions:

(a)
```
function xx = sumcos(f, X, fs, dur)
  % SUMCOS Function to synthesize a sum of cosine waves
  % usage:
  % xx = sumcos(f, X, fs, dur)
  % f = vector of frequencies (these could be negative or positive)
  % X = vector of complex exponentials:  Amp*e^(j*phase)
  % fs = the sampling rate in Hz
  % dur = total time duration of signal
  %
  % Note:  f and X must be the same length.
  % X(1) corresponds to frequency f(1),
  % X(2) corresponds to frequency f(2), etc.

  t = 0:1/fs:dur;
  xx = exp(j*2*pi*f'*t); % this make a matrix were the columns

  % correspond to different times and the
  % rows correspond to the different
  % frequency components.

  xx = real(X * xx); % this sums all the components up to return
```
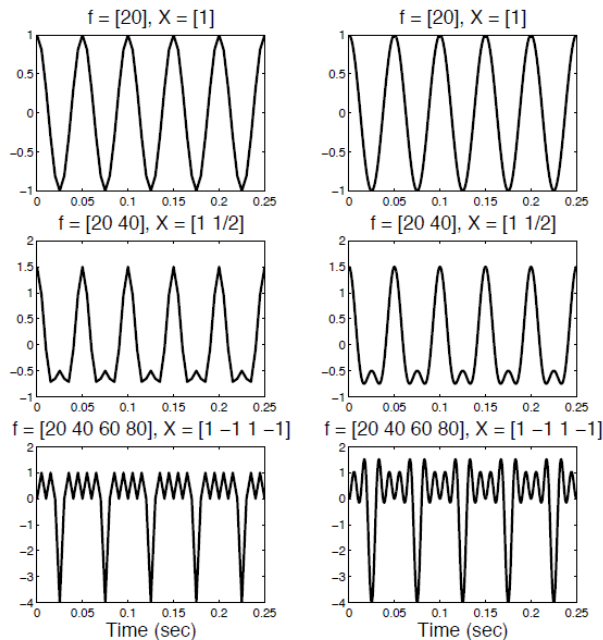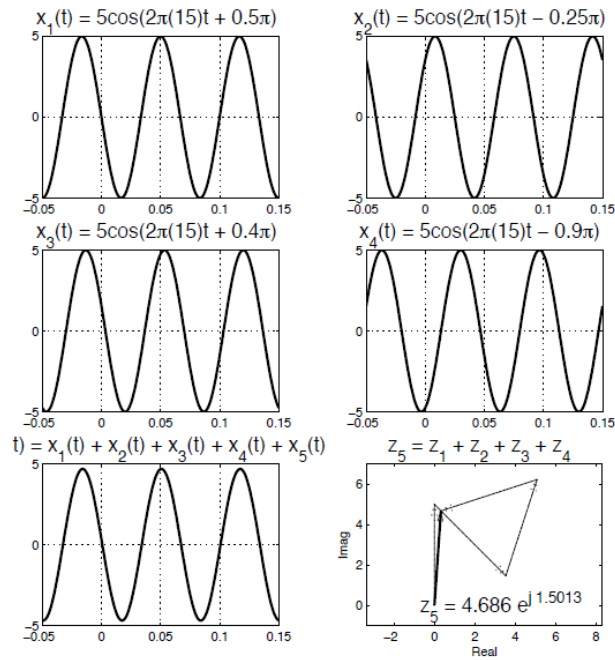

Figure 5: Testing sumcos.m - C.2.2.2

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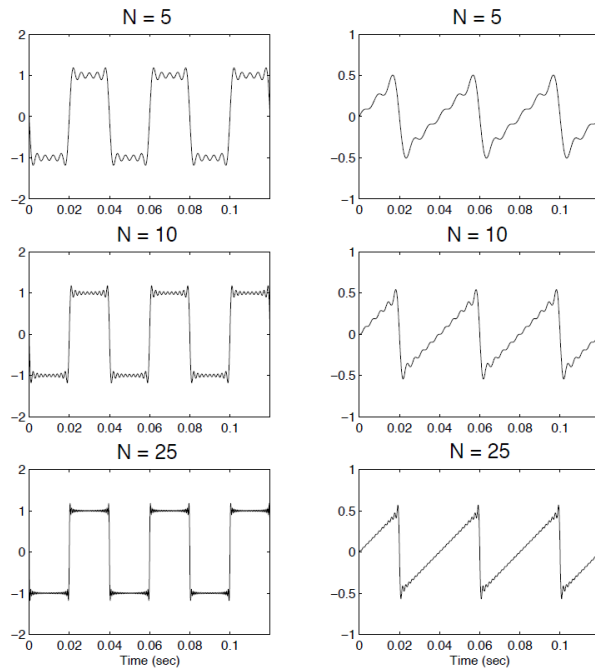Figure 6: Addition of Sinusoids - C.2.3.2

Figure 7: Periodic Waveforms - C.2.4

7. **Additional problem (for MAS.510): Listening to Sinusoids**

We have seen many examples of sinusoids where amplitude, frequency, and phase remain constant. However, these parameters can be varied, often with interesting and surprising results. In this problem, you will use MATLAB to explore these possibilities.

Start with a basic sinusoid of the following form:

$$x(t) = A(t)sin(f \cdot 2\pi t + g(t)) \tag{1}$$

(a) In MATLAB, use the equation above to create a 500 Hz tone of constant amplitude ($A(t) = 1$) and constant phase ($g(t) = 0$) lasting two seconds. Then alter its frequency so that it rises linearly from 500 Hz to 2500 Hz. Listen to the resulting signal using the MATLAB function sound. Plot enough of the signal so that the change in frequency is visible. This signal is called a *chirp*. (Note: use the sound command at the Matlab prompt. And when you are selecting a sampling rate try to use 7500 Hz, and see if your results sound better.

(b) Start again with the 500Hz tone. This time, vary the *amplitude* using the following equation:

$$A(t) = sin(N2\pi t) \tag{2}$$

Keep the phase constant (i.e. $g(t) = 0$). Vary N with a few values from 1 to 500 and listen to the result. How does the sound change with different values of N? Plot enough of the signal so that the change to the signal is apparent.

(c) Return to the constant amplitude signal ($A(t) = 1$). This time change the phase to be the following:

$$g(t) = cos(M2\pi t) \tag{3}$$

Try several values of M between 10 and 700 and listen to the result. How does the sound change with different values of M? Plot enough of the signal so that the change in the signal is apparent.

(d) Oftentimes, the modifications themselves contain the signal of interest, while the original sinusoid becomes what is called the *carrier signal*, with frequency f. The changes applied in parts (b) and (c) are called *Amplitude Modulation* (AM) and *Frequency Modulation* (FM), respectively. Where else have you heard these terms? Give some examples of your favorite values of f.
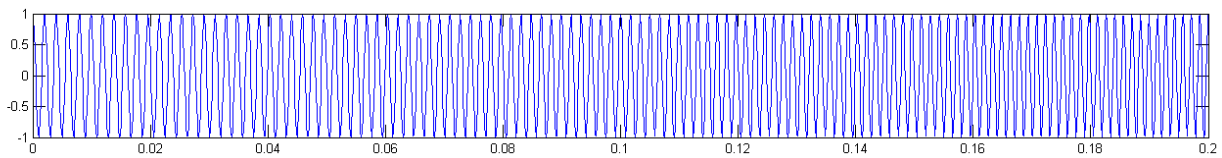
Solutions:

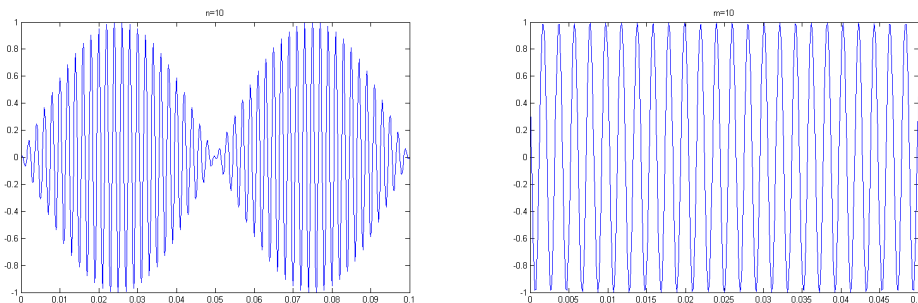Figure 8: The change in frequency is clearly audible and even visible at the beginning at the chirp
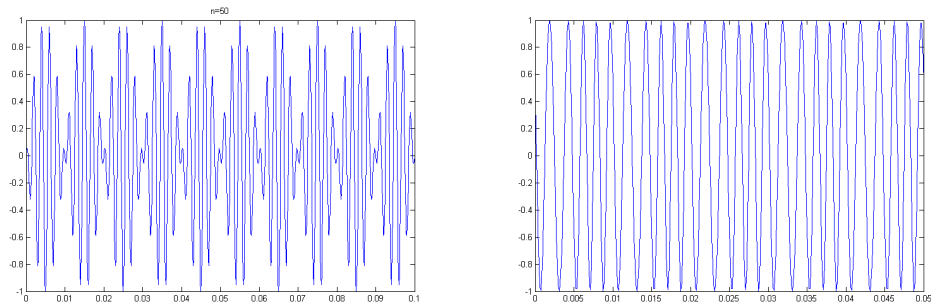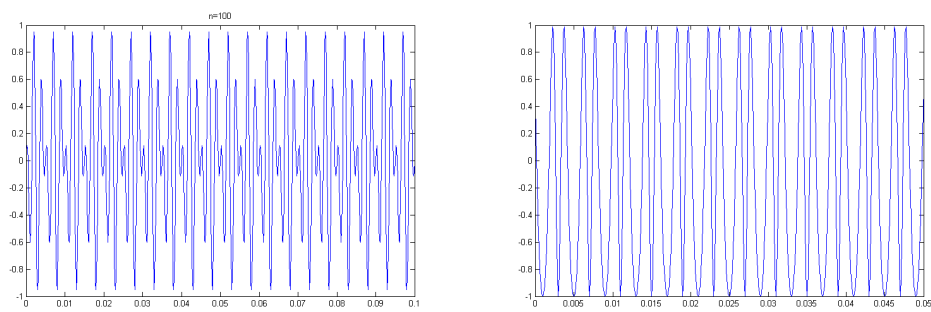

Figure 9: N=20, M= 10

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Figure 10: N=50, M=100

Figure 11: N=100, M=250