# Shift Invariance Example

## 1 Problem

Determine whether the system describe by the following relation is shift-invariant:

$$y(t) = x(sin(t))$$

## 2 Solution

Let $x_2(t) = x_1(t - T)$
To show shift-invariance holds for this system, we want to show that $y_2(t) = y_1(t - T)$

Recall that in words, this is equivalent to saying that if we delay our input to the system, we expect to see a delayed output. So here, $x_1(t)$ is our original input and $y_1(t)$ is our original output. Then our new input $x_2(t)$ is a delayed version of $x_1(t)$, so if the system is shift-invariant, we expect to see the new output $y_2(t)$ is just a delayed version of $y_1(t)$

We have $y_1(t) = x_1(sin(t))$
We replace $t$ with $t - T$ to obtain:
$y_1(t - T) = x_1(sin(t - T))$

We also have $y_2(t) = x_2(sin(t))$
We replace $t$ with $sin(t)$ to obtain:
$x_2(sin(t)) = x_1(sin(t) - T) \Rightarrow y_2(t) = x_1(sin(t) - T)$

From this, we see that $y_2(t) \neq y_1(t - T)$
Thus, the system is **not** shift-invariant!