MAS 160/510 Recitation 2

Friday February 17, 2012

1. **Sinusoids and periodicity:** Just because it looks like a sine doesn't make it periodic!

   (a) $x(t) = \sin(t^2)$
   Not periodic!

   (b) $x[n] = \cos(7.7\pi n) = \cos(6\pi n + 1.7\pi n) = \cos(1.7\pi n)$
   $\Rightarrow \frac{17\pi}{10} T = 2n\pi \Rightarrow T = 20$

   (c) $x[n] = \sin(5n)$
   Also not periodic!

2. **Integration!**

   We have represented a period function with period $T_0 = 1/f_0$:

$$x(t) = X_0 + \Re e\{\sum_{k=1}^{\infty} X_k e^{j2\pi k f_0 t}\} \quad (1)$$

   We know that the coefficients can be found using the following equations:

$$X_0 = \frac{1}{T_0} \int_0^{T_0} x(t) dt \quad (2)$$

$$X_k = \frac{2}{T_0} \int_0^{T_0} x(t) e^{-j2\pi kt/T_0} dt \qquad \text{for } k \neq 0 \quad (3)$$

   We will attempt to show *why* these analysis equations work!

   Evaluate the following integral in each of the two cases:

$$\int_0^{T_0} e^{j2\pi n f_0 t} e^{-j2\pi m f_0 t} \, dt \qquad \text{for } m \neq 0$$

   (a) For $n = m$:
   $\int_0^{T_0} e^{j2\pi n f_0 t} e^{-j2\pi m f_0 t} \, dt = \int_0^{T_0} 1 \, dt = T_0$

   (b) For $n \neq m$:
   $\int_0^{T_0} e^{j2\pi n f_0 t} e^{-j2\pi m f_0 t} \, dt = \int_0^{T_0} e^{j2\pi (n-m) f_0 t} \, dt = \frac{1}{j2\pi(n-m)f_0 t} e^{j2\pi(n-m)f_0 t} \Big|_{t=0}^{T_0} = 0$

3. **More integration??!**

   (a) $\int |x| dx = \begin{cases} \int -x \, dx & \text{for } x < 0 \\ \int x \, dx & \text{for } x \geq 0 \end{cases}$
   $= \begin{cases} -x^2/2 + C & \text{for } x < 0 \\ x^2/2 + C & \text{for } x \geq 0 \end{cases}$

   (b) $\int t e^{j2\pi f t} dt = \frac{1}{j2\pi f} t e^{j2\pi f t} - \frac{1}{(j2\pi f)^2} e^{j2\pi f t}$