

Causal FIR filter

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

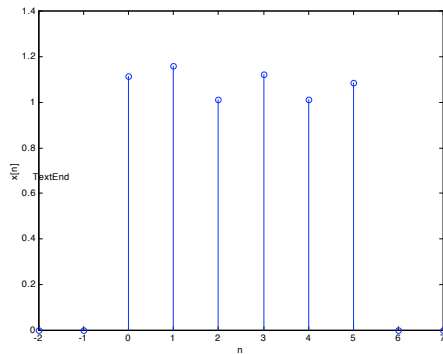
The output y at each sample n is a weighted sum of the present input, $x[n]$, and past inputs, $x[n-1]$, $x[n-2]$, ..., $x[n-M]$.

3 point average

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n=-2 -1 0 1 2 3 4 5 6 7



3 point average

causal running average or backward average

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$b_0 = \frac{1}{3} \quad b_1 = \frac{1}{3} \quad b_2 = \frac{1}{3}$$

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

difference equation

$$L=3 \quad M=L-1=2$$

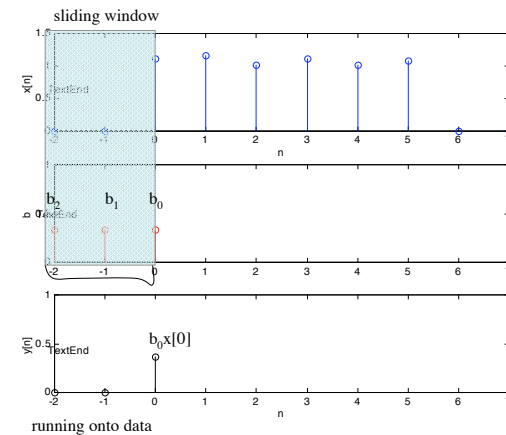
Length 3 2nd order

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[0] = \frac{1}{3} x[0] + \frac{1}{3} x[-1] + \frac{1}{3} x[-2] = \frac{1}{3} 1.11 + \frac{1}{3} 0 + \frac{1}{3} 0 = 0.36$$

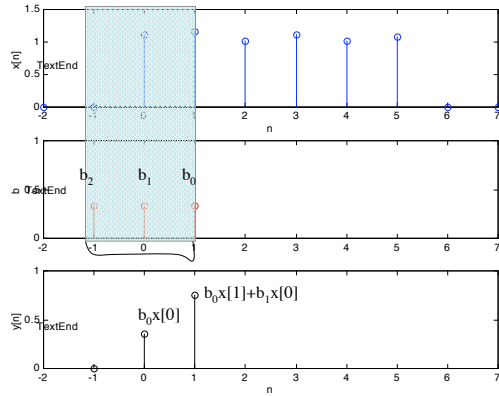


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad \mathbf{1.11} \quad \mathbf{1.16} \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[1] = \frac{1}{3}x[1] + \frac{1}{3}x[0] + \frac{1}{3}x[-1] = \frac{1}{3}1.16 + \frac{1}{3}1.11 + \frac{1}{3}0 = 0.76$$

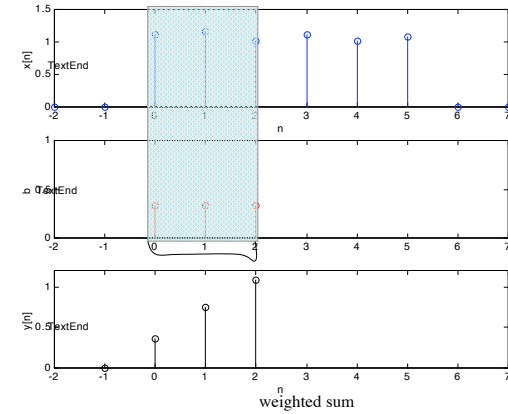


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad \mathbf{1.11} \quad \mathbf{1.16} \quad \mathbf{1.01} \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[2] = \frac{1}{3}x[2] + \frac{1}{3}x[1] + \frac{1}{3}x[0] = \frac{1}{3}1.01 + \frac{1}{3}1.16 + \frac{1}{3}1.11 = 1.09$$

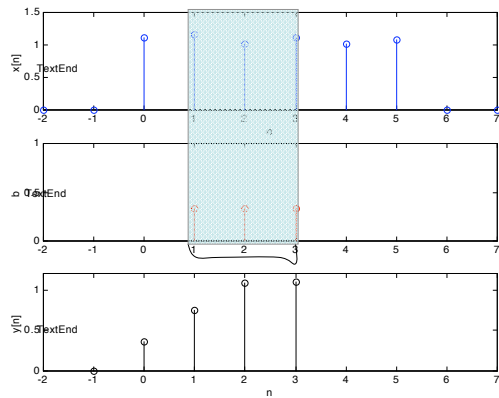


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad \mathbf{1.16} \quad \mathbf{1.01} \quad \mathbf{1.12} \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[3] = \frac{1}{3}x[3] + \frac{1}{3}x[2] + \frac{1}{3}x[1] = \frac{1}{3}1.12 + \frac{1}{3}1.01 + \frac{1}{3}1.16 = 1.10$$

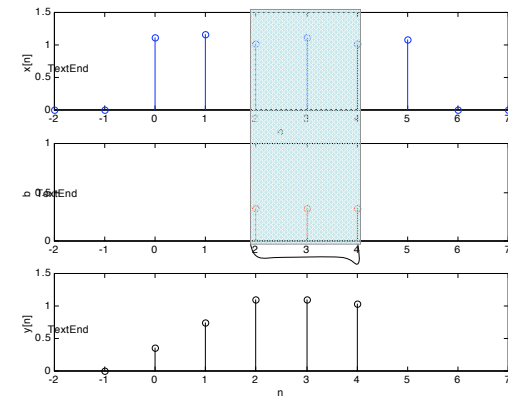


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad \mathbf{1.01} \quad \mathbf{1.12} \quad \mathbf{1.01} \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

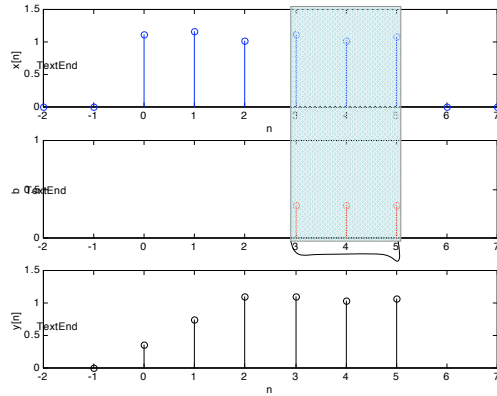
$$y[4] = \frac{1}{3}x[4] + \frac{1}{3}x[3] + \frac{1}{3}x[2] = \frac{1}{3}1.01 + \frac{1}{3}1.12 + \frac{1}{3}1.01 = 1.05$$



$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0 \ 0\}$$

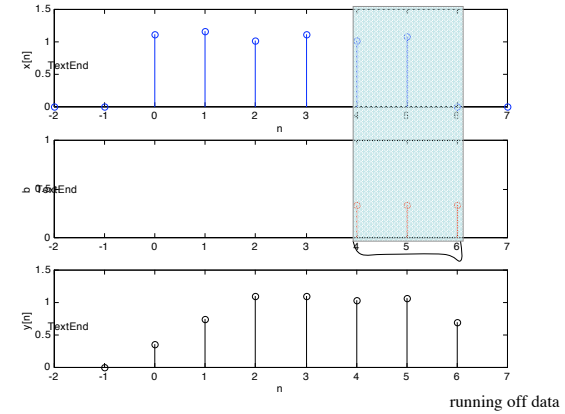
$$y[5] = \frac{1}{3}x[5] + \frac{1}{3}x[4] + \frac{1}{3}x[3] = \frac{1}{3}1.08 + \frac{1}{3}1.01 + \frac{1}{3}1.12 = 1.07$$



$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0 \ 0\}$$

$$y[6] = \frac{1}{3}x[6] + \frac{1}{3}x[5] + \frac{1}{3}x[4] = \frac{1}{3}0 + \frac{1}{3}1.08 + \frac{1}{3}1.01 = 0.70$$

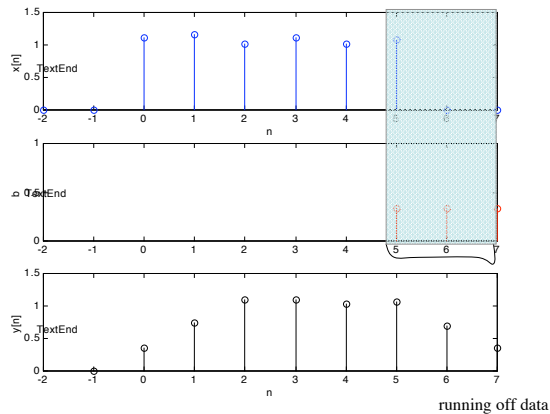


running off data

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0 \ 0\}$$

$$y[7] = \frac{1}{3}x[7] + \frac{1}{3}x[6] + \frac{1}{3}x[5] = \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}1.08 = 0.36$$

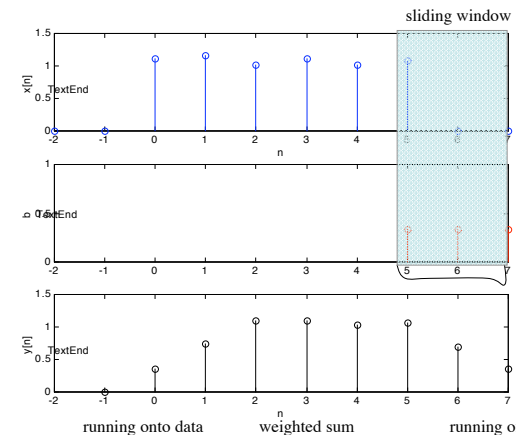


running off data

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \ 0 \ 1.11 \ 1.16 \ 1.01 \ 1.12 \ 1.01 \ 1.08 \ 0.0 \ 0.0\}$$

$$y[n] = \{0 \ 0 \ 0.36 \ 0.75 \ 1.09 \ 1.10 \ 1.05 \ 1.07 \ 0.7 \ 0.36\}$$



running onto data

weighted sum

running off data

Notice:
 $y[n]$ is longer than $x[n]$.
 $x[n]$ is M/2 delayed

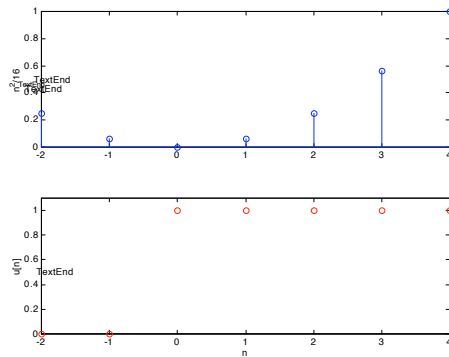
2 point difference

$$y[n] = x[n] - x[n-1]$$

$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit step function



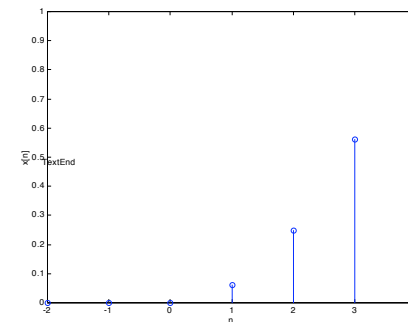
2 point difference

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unit step function



$$y[n] = x[n] - x[n-1]$$

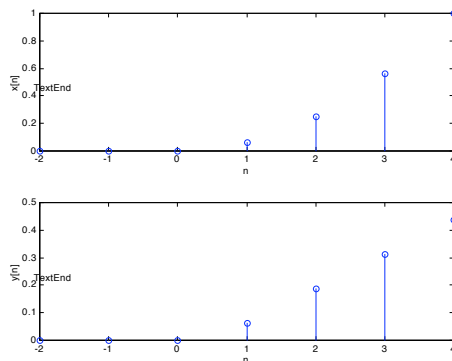
$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$x[n] = \{0 \quad 0 \quad 0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16}\}$$

$$y[n] = \{0 \quad 0 \quad 0 \quad \frac{1}{16} \quad \frac{3}{16} \quad \frac{5}{16} \quad \frac{7}{16}\} = \frac{2n-1}{16} u[n-1]$$

finite difference approximation to a derivative.

derivatives enhance noise (and high frequencies)



Impulse response

$$y[n] = \sum_{k=0}^M b_k x[n-k] \quad \text{FIR filter}$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Kronecker delta function}$$



$$y[n] = h[n] = \sum_{k=0}^M b_k \delta[n-k] \quad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & \text{otherwise} \end{cases}$$

impulse response

The impulse response is the output of the system when the input is a delta function.

Impulse response

$$h[n] = y[n] = \sum_{k=0}^M b_k \delta[n-k] \quad \delta[n-k] = \begin{cases} 1 & n=k \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} h[n] &= b_0 \delta[n-0] + b_1 \delta[n-1] + \dots + b_n \delta[n-n] + \dots + b_M \delta[M-1] \\ &= b_0 \delta[n-0] + b_1 \delta[n-1] + \dots + b_n \delta[0] + \dots + b_M \delta[M-1] \\ &= b_0 0 + b_1 0 + \dots + b_n 1 + \dots + b_M 0 \quad \delta[z] = \begin{cases} 1 & z=0 \\ 0 & \text{otherwise} \end{cases} \\ &= b_n \quad \text{impulse response} \end{aligned}$$

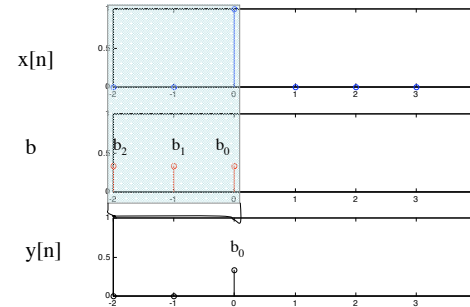
The impulse response is just the filter coefficients.
Finite length filter, finite impulse response (FIR).

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Delta function}$$

$$y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2]$$



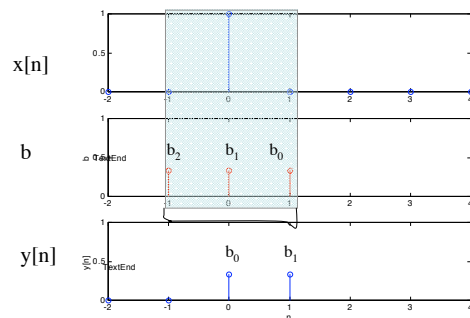
$$\begin{aligned} y[0] &= \frac{1}{3} \delta[0] + \frac{1}{3} \delta[-1] + \frac{1}{3} \delta[-2] \\ &= \frac{1}{3} 1 + \frac{1}{3} 0 + \frac{1}{3} 0 = \frac{1}{3} \end{aligned}$$

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Delta function}$$

$$y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2]$$



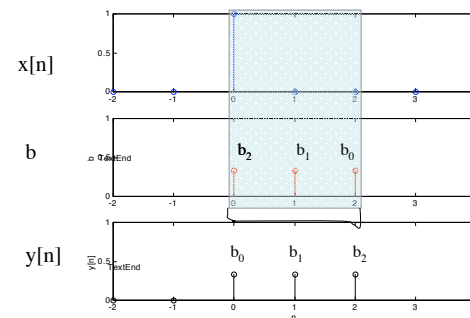
$$\begin{aligned} y[1] &= \frac{1}{3} \delta[1] + \frac{1}{3} \delta[0] + \frac{1}{3} \delta[-1] \\ &= \frac{1}{3} 0 + \frac{1}{3} 1 + \frac{1}{3} 0 = \frac{1}{3} \end{aligned}$$

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Delta function}$$

$$y[n] = \frac{1}{3} \delta[n] + \frac{1}{3} \delta[n-1] + \frac{1}{3} \delta[n-2]$$



$$\begin{aligned} y[2] &= \frac{1}{3} \delta[2] + \frac{1}{3} \delta[1] + \frac{1}{3} \delta[0] \\ &= \frac{1}{3} 0 + \frac{1}{3} 0 + \frac{1}{3} 1 = \frac{1}{3} \end{aligned}$$

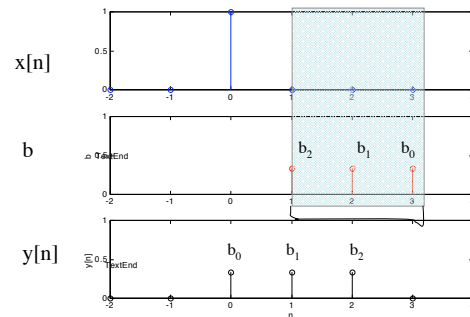
Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$$

Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



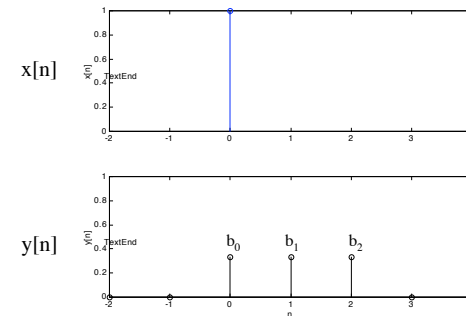
$$\begin{aligned} y[3] &= \frac{1}{3}\delta[3] + \frac{1}{3}\delta[2] + \frac{1}{3}\delta[1] \\ &= \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}0 = 0 \end{aligned}$$

Impulse response of 3 pt. average

$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

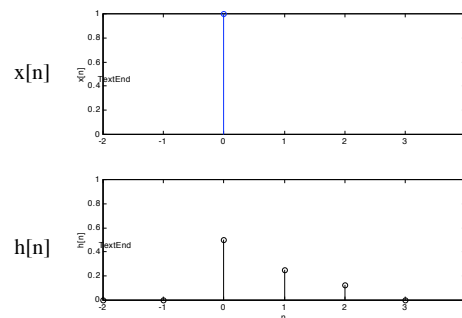
$$x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

$$\begin{aligned} h[n] = y[n] \big|_{x[n] = \delta[n]} &= \{0 \quad 0 \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad 0 \quad 0\} \\ &= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\} \end{aligned}$$



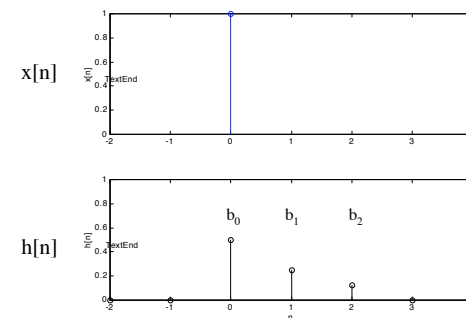
Coefficients from impulse response

$$\begin{aligned} x[n] = \delta[n] &= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\} \\ n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \end{aligned} \quad \longrightarrow \quad \boxed{y[n] = \sum_{k=0}^M b_k x[n-k]} \quad \longrightarrow \quad \begin{aligned} h[n] &= y[n] \big|_{x[n] = \delta[n]} \\ &= \{0 \quad 0 \quad \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \quad 0 \quad 0\} \end{aligned}$$



Coefficients from impulse response

$$\begin{aligned} x[n] = \delta[n] &= \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\} \\ n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \end{aligned} \quad \longrightarrow \quad \boxed{y[n] = \sum_{k=0}^M b_k x[n-k]} \quad \longrightarrow \quad \begin{aligned} h[n] &= y[n] \big|_{x[n] = \delta[n]} \\ &= \{0 \quad 0 \quad \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \quad 0 \quad 0\} \\ &= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\} \end{aligned}$$

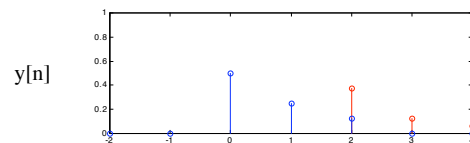
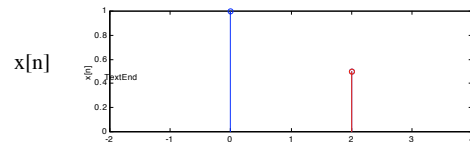


Response from 2 impulses

$$x[n] = \delta[n] + 0.5\delta[n-2] \longrightarrow \begin{cases} y[n] = \sum_{k=0}^M b_k x[n-k] \\ \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \end{cases} \longrightarrow \left\{ 0 \quad 0 \quad \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} + \frac{4}{16} \quad \frac{2}{8} \quad \frac{1}{16} \right\}$$

$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$$y[n] = \{0 \quad 0 \quad b_0 x[0] \quad b_1 x[0] \quad b_2 x[0] + b_0 x[2] \quad b_1 x[3] \quad b_2 x[4]\}$$

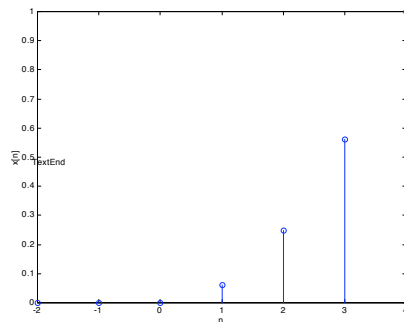


Sum the responses of each impulse

$$x[n] = \frac{n^2}{16} \cdot u[n] \quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$x[n] = \left\{ 0 \quad 0 \quad 0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \right\}$$

$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$



$$x[n] = 0 \cdot \delta[n] + \frac{1}{16} \cdot \delta[n-1] + \frac{4}{16} \cdot \delta[n-2] + \frac{9}{16} \cdot \delta[n-3] + \frac{16}{16} \cdot \delta[n-4]$$

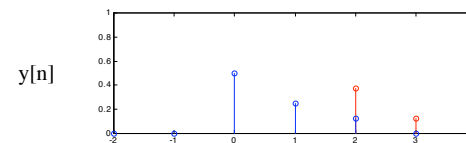
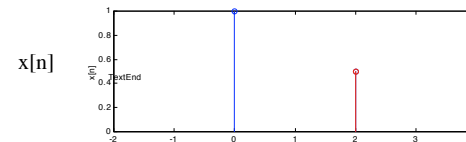
Any discrete signal be thought of a weighted sum of delayed impulses

Response from 2 impulses

$$x[n] = \delta[n] \longrightarrow \begin{cases} y[n] = \sum_{k=0}^M b_k x[n-k] \\ h[n] = \{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\} \end{cases} \longrightarrow \left\{ 0 \quad 0 \quad \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} + \frac{4}{16} \quad \frac{2}{8} \quad \frac{1}{16} \right\}$$

$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$$y[n] = \{0 \quad 0 \quad h[0]x[0] \quad h[1]x[0] \quad h[2]x[0] + h[0]x[2] \quad h[1]x[3] \quad h[2]x[4]\}$$



$$y[2] = h[0]x[2-0] + h[1]x[2-1] + h[2]x[2-2]$$

$$y[n] = \sum_{k=0}^3 h[k]x[n-k]$$

Convolution sum

Convolution

$$y[n] = \sum_{k=0}^M b_k x[n-k] \quad \text{FIR filter}$$

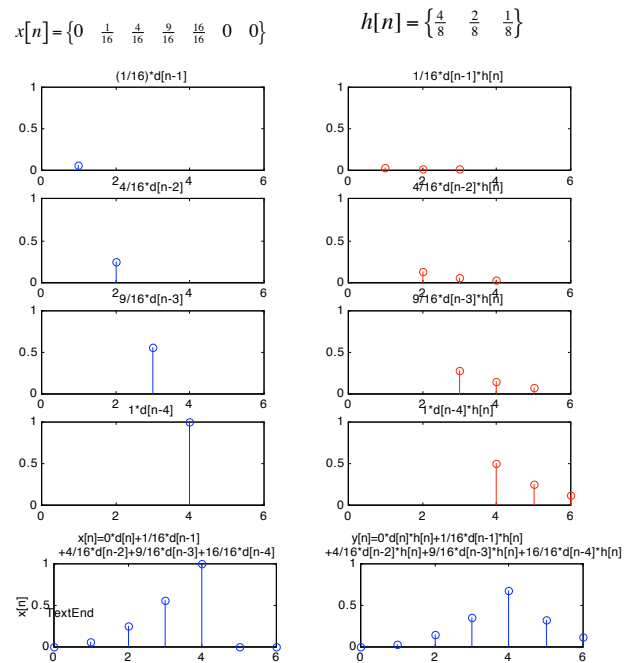
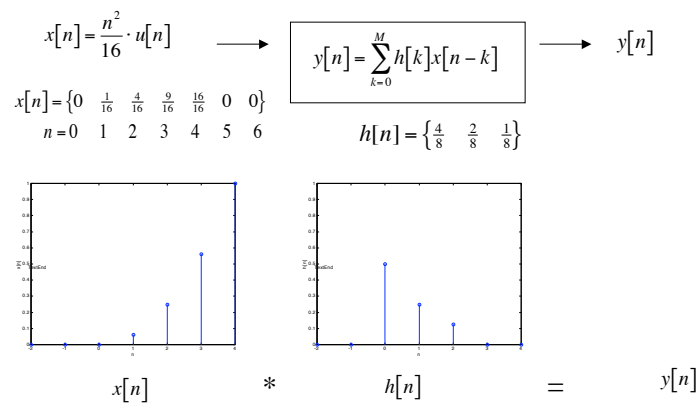
$$h[n] = y[n] \Big|_{x[n] = \delta[n]} = \begin{cases} b_n & n = 0, 1, \dots, M \\ 0 & \text{otherwise} \end{cases} \quad \text{impulse response}$$

$$y[n] = \sum_{k=0}^M h[k]x[n-k] \quad \text{convolution sum}$$

or

$$y[n] = h * x$$

The output $y[n]$ is equal to the input $x[n]$ convolved with the unit impulse response $h[n]$.



$$x[n] = \left\{ \begin{matrix} 0 & \frac{1}{16} & \frac{4}{16} & \frac{9}{16} & \frac{16}{16} & 0 & 0 \\ n=0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \right\} \quad h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]	4/8	2/8	1/8				
=====							
	0	0	0				

$$x[n] = \left\{ \begin{matrix} 0 & \frac{1}{16} & \frac{4}{16} & \frac{9}{16} & \frac{16}{16} & 0 & 0 \\ n=0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \right\} \quad h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]		4/8	2/8	1/8			
=====							
0		0	0				
		4/128	2/128	1/128			

$$x[n] = \{0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \quad 0 \quad 0\}$$

$$h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]			4/8	2/8	1/8		
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
=====							

$$x[n] = \{0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \quad 0 \quad 0\}$$

$$h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]				4/8	2/8	1/8	
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
=====							

$$x[n] = \{0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \quad 0 \quad 0\}$$

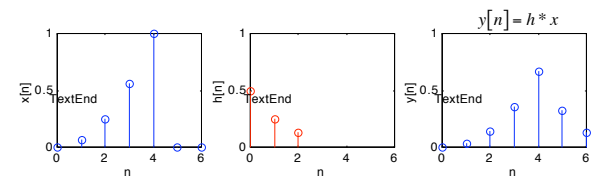
$$h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]					4/8	2/8	1/8
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
					64/128	32/128	16/128
=====							
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128

$$x[n] = \{0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \quad 0 \quad 0\}$$

$$h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

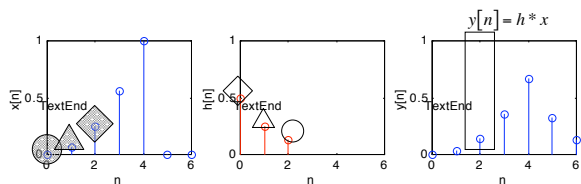
x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]	4/8	2/8	1/8				
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
					64/128	32/128	16/128
=====							
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128



n	0	1	2	3	4	5	6
x[n]	0	● 1/16	▲ 4/16	◆ 9/16	16/16	0	0
h[n]	4/8	◇ 2/8	△ 1/8	○			
	0 ◇	0 ●	0 ○				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
					64/128	32/128	16/128
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128

$$y[2] = h[0]x[2-0] + h[1]x[2-1] + h[2]x[2-2]$$

◇ ◆ △ ▲ ○ ●



$$x[n] = \left\{ \begin{matrix} 0 & \frac{1}{16} & \frac{4}{16} & \frac{9}{16} & \frac{16}{16} & 0 & 0 \end{matrix} \right\}_{n=0}^6 \quad h[n] = \left\{ \frac{4}{8}, \frac{2}{8}, \frac{1}{8} \right\}_{n=0}^3 \quad L=3, M=L-1=2$$

$$x[n] = 0 \cdot \delta[n] + \frac{1}{16} \cdot \delta[n-1] + \frac{4}{16} \cdot \delta[n-2] + \frac{9}{16} \cdot \delta[n-3] + \frac{16}{16} \cdot \delta[n-4]$$

$$h[n] = \frac{4}{8} \cdot \delta[n] + \frac{2}{8} \cdot \delta[n-1] + \frac{1}{8} \cdot \delta[n-2]$$

$$y[n] = \sum_{k=0}^M h[k]x[n-k] \quad \text{convolution sum}$$

$$y[0] = h[0]x[0-0] + h[1]x[0-1] + h[2]x[0-2] = \frac{4}{8} \cdot 0 + \frac{2}{8} \cdot 0 + \frac{1}{8} \cdot 0 = 0$$

$$y[1] = h[0]x[1-0] + h[1]x[1-1] + h[2]x[1-2] = \frac{4}{8} \cdot \frac{1}{16} + \frac{2}{8} \cdot 0 + \frac{1}{8} \cdot 0 = \frac{4}{128}$$

$$y[2] = h[0]x[2-0] + h[1]x[2-1] + h[2]x[2-2]$$

$$y[2] = \sum_{k=0}^2 h[k]x[2-k]$$

$$y[n] = \sum_{k=0}^M h[k]x[n-k] \quad \text{convolution sum}$$

$$x[n] = \left\{ \begin{matrix} 0 & \frac{1}{16} & \frac{4}{16} & \frac{9}{16} & \frac{16}{16} & 0 & 0 \end{matrix} \right\}_{n=0}^6 \quad h[n] = \left\{ \frac{4}{8}, \frac{2}{8}, \frac{1}{8} \right\}_{n=0}^3 \quad L=3, M=L-1=2$$

$$x[n] = 0 \cdot \delta[n] + \frac{1}{16} \cdot \delta[n-1] + \frac{4}{16} \cdot \delta[n-2] + \frac{9}{16} \cdot \delta[n-3] + \frac{16}{16} \cdot \delta[n-4]$$

$$h[n] = \frac{4}{8} \cdot \delta[n] + \frac{2}{8} \cdot \delta[n-1] + \frac{1}{8} \cdot \delta[n-2]$$

$$y[n] = \sum_{k=0}^M h[k]x[n-k] \quad \text{convolution sum}$$

$$y[0] = h[0]x[0-0] + h[1]x[0-1] + h[2]x[0-2] = \frac{4}{8} \cdot 0 + \frac{2}{8} \cdot 0 + \frac{1}{8} \cdot 0 = 0$$

$$y[1] = h[0]x[1-0] + h[1]x[1-1] + h[2]x[1-2] = \frac{4}{8} \cdot \frac{1}{16} + \frac{2}{8} \cdot 0 + \frac{1}{8} \cdot 0 = \frac{4}{128}$$

$$y[2] = h[0]x[2-0] + h[1]x[2-1] + h[2]x[2-2] = \frac{4}{8} \cdot \frac{4}{16} + \frac{2}{8} \cdot \frac{1}{16} + \frac{1}{8} \cdot 0 = \frac{18}{128}$$

$$y[3] = h[0]x[3-0] + h[1]x[3-1] + h[2]x[3-2] = \frac{4}{8} \cdot \frac{9}{16} + \frac{2}{8} \cdot \frac{4}{16} + \frac{1}{8} \cdot \frac{1}{16} = \frac{45}{128}$$

$$y[4] = h[0]x[4-0] + h[1]x[4-1] + h[2]x[4-2] = \frac{4}{8} \cdot \frac{16}{16} + \frac{2}{8} \cdot \frac{9}{16} + \frac{1}{8} \cdot \frac{4}{16} = \frac{86}{128}$$

$$y[5] = h[0]x[5-0] + h[1]x[5-1] + h[2]x[5-2] = \frac{4}{8} \cdot 0 + \frac{2}{8} \cdot \frac{16}{16} + \frac{1}{8} \cdot \frac{9}{16} = \frac{41}{128}$$

$$y[6] = h[0]x[6-0] + h[1]x[6-1] + h[2]x[6-2] = \frac{4}{8} \cdot 0 + \frac{2}{8} \cdot 0 + \frac{1}{8} \cdot \frac{16}{16} = \frac{16}{128}$$

$$x[n] = \{0 \quad \frac{1}{16} \quad \frac{4}{16} \quad \frac{9}{16} \quad \frac{16}{16} \quad 0 \quad 0\}$$

$$h[n] = \left\{ \frac{4}{8} \quad \frac{2}{8} \quad \frac{1}{8} \right\}$$

$x[n]$	0	1/16	4/16	9/16	16/16	0	0
$h[n]$	4/8	2/8	1/8				

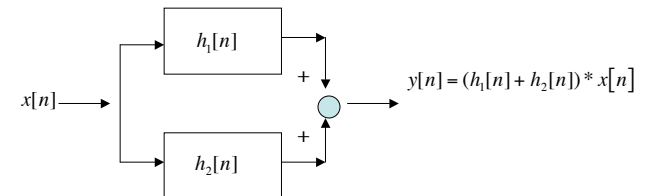
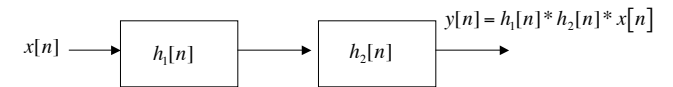
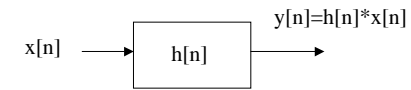
$x[n]*h[n]$	0	4/128	18/128	45/128	86/128	41/128	16/128
	0	0.3125	0.1406	0.3516	0.6719	0.3203	0.125

MATLAB

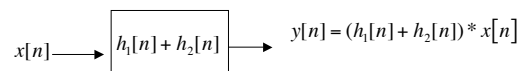
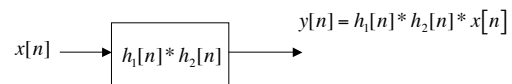
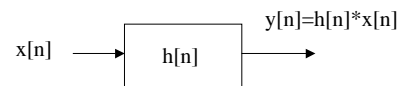
```
>>conv([4/8, 2/8, 1/8], [0, 1/16, 4/16, 9/16, 16/16])
```

```
ans =  
    0    0.0312    0.1406    0.3516    0.6719    0.3203    0.1250
```

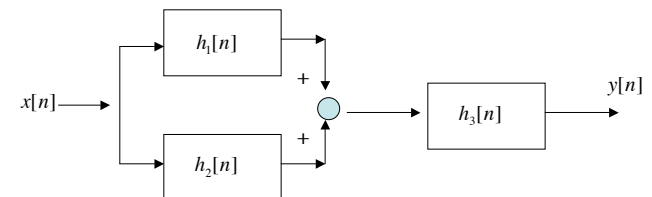
LTI Systems



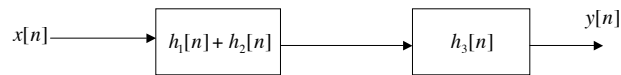
LTI Systems



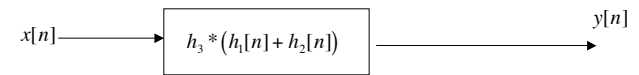
LTI Systems



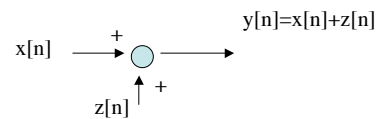
LTI Systems



LTI Systems

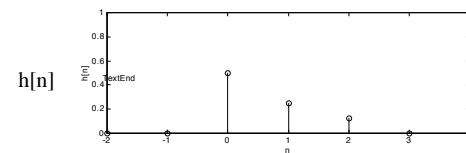
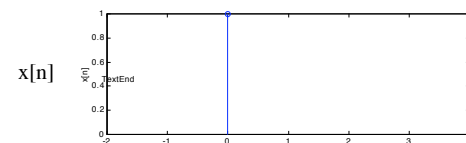


Block Diagrams



Block Diagrams: Direct Form

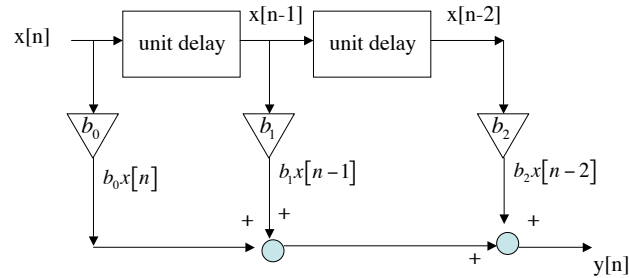
$$\begin{aligned}
 x[n] &= \delta[n] \\
 &= \{0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0\} \\
 n &= -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \\
 &L=3, M=L-1=2
 \end{aligned}
 \rightarrow
 \begin{aligned}
 y[n] &= \sum_{k=0}^M b_k x[n-k] \\
 \{b_0, b_1, b_2\} &= \left\{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\right\}
 \end{aligned}
 \rightarrow
 \begin{aligned}
 h[n] &= y[n]_{|x[n]=\delta[n]} \\
 &= \left\{0 \ 0 \ \frac{4}{8} \ \frac{2}{8} \ \frac{1}{8} \ 0 \ 0\right\} \\
 &= \left\{0 \ 0 \ b_0 \ b_1 \ b_2 \ 0 \ 0\right\}
 \end{aligned}$$



Block Diagrams: Direct Form

$$\begin{aligned}
 x[n] &= \delta[n] \\
 &= \{0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0\} \\
 n &= -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \\
 &\quad L=3, M=L-1=2
 \end{aligned}
 \rightarrow
 \begin{aligned}
 y[n] &= \sum_{k=0}^M b_k x[n-k] \\
 \{b_0, b_1, b_2\} &= \left\{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\right\}
 \end{aligned}
 \rightarrow
 \begin{aligned}
 h[n] &= y[n]_{[x[n]=\delta[n]]} \\
 &= \left\{0 \ 0 \ \frac{4}{8} \ \frac{2}{8} \ \frac{1}{8} \ 0 \ 0\right\} \\
 &= \left\{0 \ 0 \ b_0 \ b_1 \ b_2 \ 0 \ 0\right\}
 \end{aligned}$$

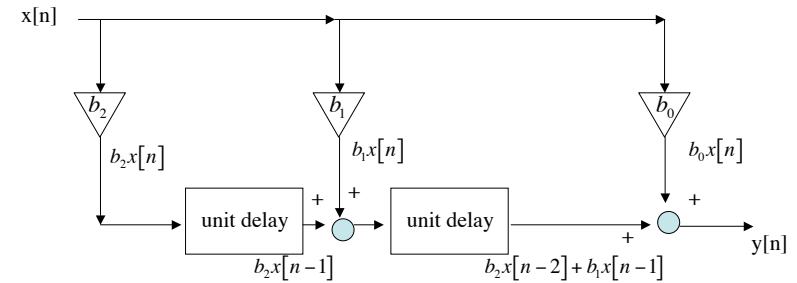
$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$



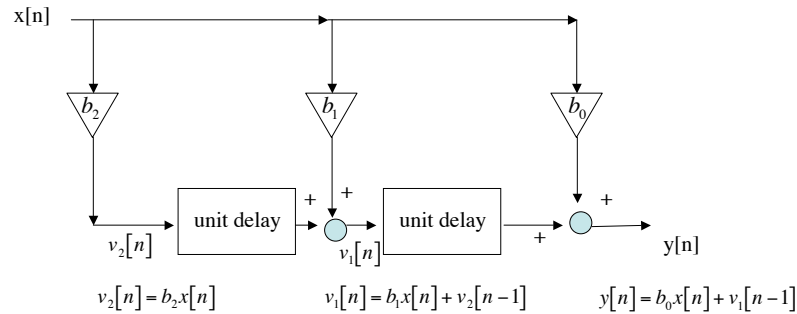
Block Diagrams: Transpose Form

$$\begin{aligned}
 x[n] &= \delta[n] \\
 &= \{0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0\} \\
 n &= -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4
 \end{aligned}
 \rightarrow
 \begin{aligned}
 y[n] &= \sum_{k=0}^M b_k x[n-k] \\
 \{b_0, b_1, b_2\} &= \left\{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\right\}
 \end{aligned}
 \rightarrow
 \begin{aligned}
 h[n] &= y[n]_{[x[n]=\delta[n]]} \\
 &= \left\{0 \ 0 \ \frac{4}{8} \ \frac{2}{8} \ \frac{1}{8} \ 0 \ 0\right\} \\
 &= \left\{0 \ 0 \ b_0 \ b_1 \ b_2 \ 0 \ 0\right\}
 \end{aligned}$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

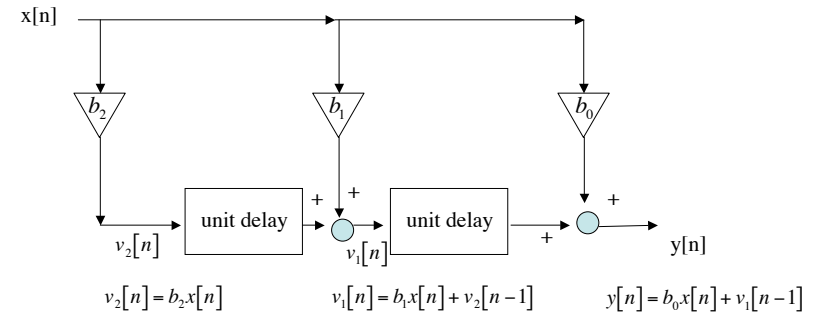


Block Diagrams to Difference Equations



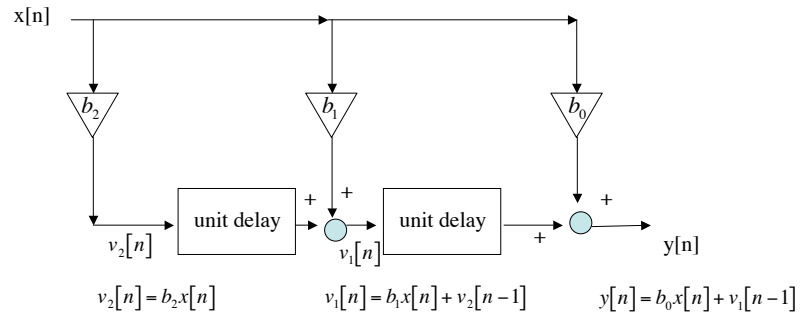
$$\begin{aligned}
 y[n] &= b_0 x[n] + v_1[n-1] \\
 v_1[n] &= b_1 x[n] + v_2[n-1] \\
 v_2[n] &= b_2 x[n]
 \end{aligned}$$

Block Diagrams to Difference Equations



$$\begin{aligned}
 y[n] &= b_0 x[n] + v_1[n-1] \\
 v_1[n-1] &= b_1 x[n-1] + v_2[n-2] \\
 v_2[n-2] &= b_2 x[n-2]
 \end{aligned}$$

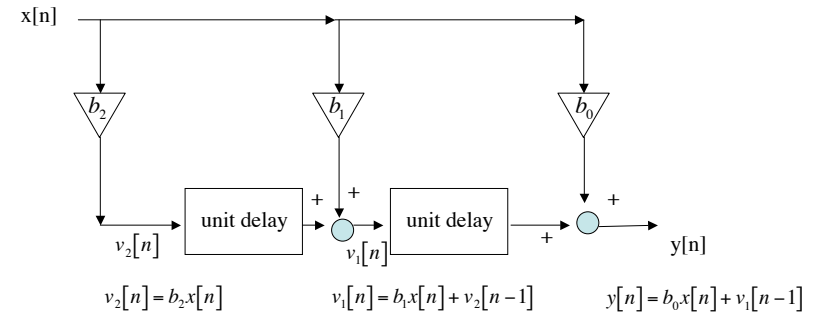
Block Diagrams to Difference Equations



$$y[n] = b_0 x[n] + v_1[n-1]$$

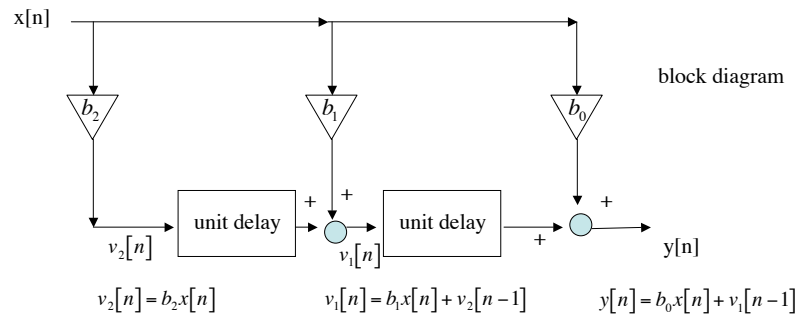
$$v_1[n-1] = b_1 x[n-1] + b_2 x[n-2]$$

Block Diagrams to Difference Equations



$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Block Diagrams to Difference Equations

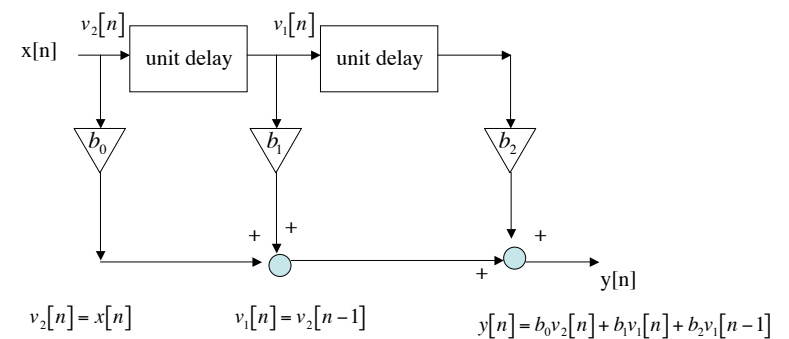


$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \quad \text{difference equation}$$

$$h[n] = b_0 \delta[n] + b_1 \delta[n-1] + b_2 \delta[n-2] \quad \text{impulse response}$$

equivalent ways
of describing system

Block Diagrams to Difference Equations

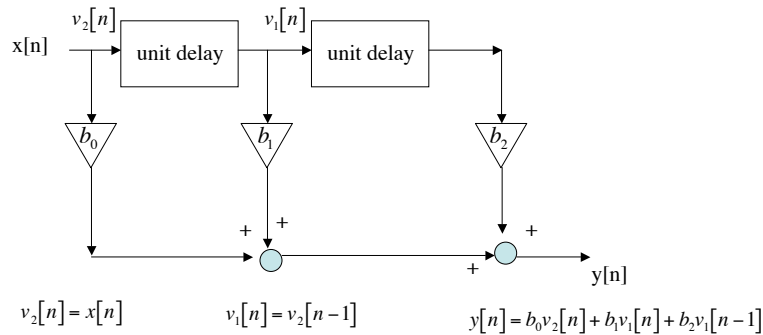


$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$

$$v_1[n] = v_2[n-1]$$

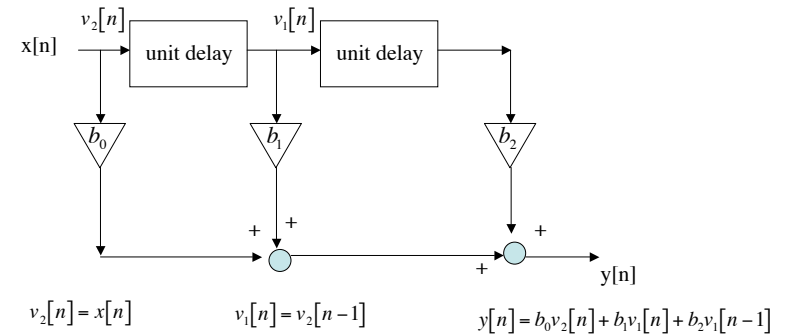
$$v_2[n] = x[n]$$

Block Diagrams to Difference Equations



$$\begin{aligned}
 y[n] &= b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1] \\
 v_2[n] &= x[n] \quad v_1[n-1] = v_2[n-2] \\
 v_2[n-2] &= x[n-2] \\
 v_1[n] &= v_2[n-1] \\
 v_2[n-1] &= x[n-1]
 \end{aligned}$$

Block Diagrams to Difference Equations



$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Homework:

p5_1: $y(n) := \frac{1}{L} \left[\sum_{k=0}^{L-1} a^{n-k} \cdot u(n-k) \right]$

hint: $\sum_{k=M}^N \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1 - \alpha}$

$$\frac{1}{L} \left[\sum_{n-z=0}^{L-1} a^z \cdot u(z) \right]$$

$$\frac{1}{L} \left[\sum_{z=n}^{n-(L-1)} a^z \cdot u(z) \right]$$

p5_6: FIR & delays
FIR and single delay

$$y[n] = ax[n] + bx[n-1]$$

L-point running average
for input sequence
 $x[n] = a^n u[n]$, $n \geq 0$

let $z = n - k$
 $k = n - z$

remember $n \geq 0$

3 point average

$$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2] \quad x[n] = \sin(2\pi n/15) \cdot u[n] \quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

