MAS.836 Sensor Technologies for Interactive Environments
Problem Set 3: FSR, Strain Gauge, and Piezo Circuits

The purpose of this problem set is to familiarize you with the most common forms of pressure and force measurements. The circuits you develop herein will also be directly relevant to other sensors, as FSRs, strain gauges, and piezos cover the range of impedances you will typically encounter. Piezo sensors are high impedance and very capacitive, FSRs are resistive with a wide range of resistance, and strain gauges are extremely low impedance sources.

Force-Sensitive Resistors

Force-sensitive resistors, or FSRs, are a fairly inexpensive and easy to implement means of measuring an applied force. They vary with time, temperature, and loading conditions, which make them inaccurate and unrepeatable, but for prototyping an idea quickly, they can be a good solution.

Strain Gauges

Strain gauges can be even less expensive than FSRs, but they also need to be mounted to some mechanical device (typically some kind of beam) that will convert the applied force to a strain. This, along with requiring more complicated conditioning electronics, makes them more difficult to work with. They are, however, many orders of magnitude more accurate and repeatable.

A strain gauge's sensitivity to the applied strain is called the *gauge factor* (S_e). The gauge factor will determine the change in resistance as a function of the applied strain according to equation 1. R_s is the resistance with applied strain, R_0 is the neutral resistance, S_e is the gauge factor, and e is the applied strain.

$$R_s = R_0(1 + S_e \cdot e) \tag{1}$$

Piezo Films

The most common form of piezoelectric sensor is a PVDF film. They are inexpensive to make and can be easily applied to many different surfaces. Just as with strain gauges, the piezo film needs to be applied to some other material which will translate the applied force into a strain in the piezo film. The piezo film will then produce a voltage proportional to this applied strain. The voltage produced will be a function of the direction of the strain, the Young's modululs of the material, the dielectric constant of the material, the piezoelectric coefficients, and the geometry of the piezo film. A detailed explanation of these relations can be found in MSI's technical guide (`https://www.sparkfun.com/datasheets/Sensors/Flex/MSI-techman.pdf`). Pay close attention to pages 3, 15-19, and 27-29.

Piezo films have many advantages in terms of ease of use, cost, and longevity, but they are not capable of maintaining a DC signal, as they have internal leakage which will drain off the charge induced by the applied strain. Furthermore, they have a high source impedance, making them susceptible to picking up noise. This is further complicated by the fact that they exhibit responses to both light and heat, so good bandpass filters should be used with piezo films.

1 Force-Sensitive Resistor Amplifiers

The graph in figure 1 is the force/resistance curve of the Interlink FSR that is included in your lab kit. The right axis is in kilo-ohms, and the left axis is in units of conductance (microsiemens.) Note that the right-hand scale is not linear, as the FSR travels through many decades of resistance over the 1 kg force range.

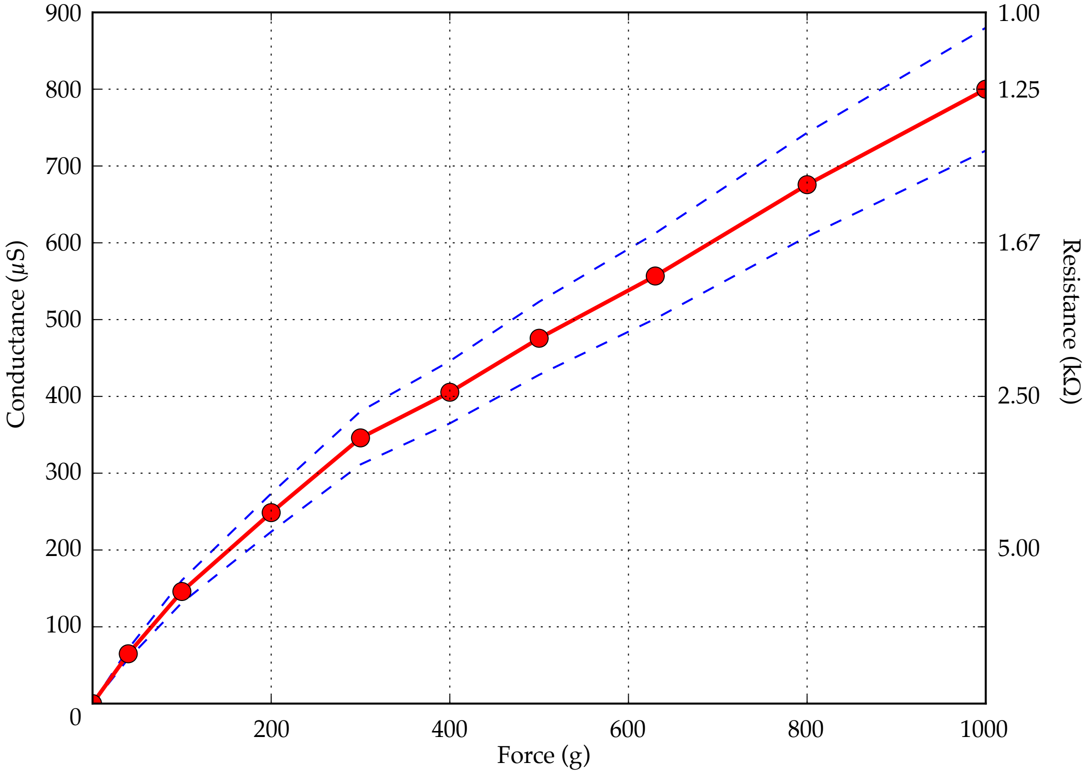

Figure 1: FSR response curve

Questions

1. Using the force/resistance relationship from figure 1, plot V_0 against applied force for the circuit in figure 2.

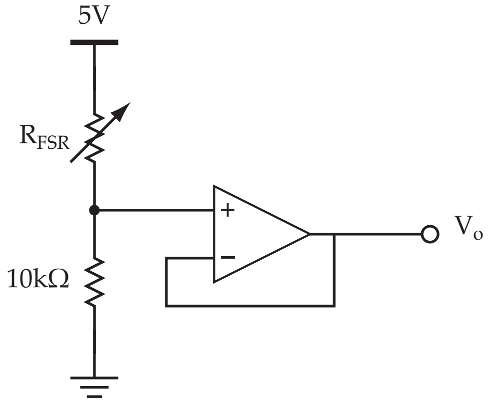

Figure 2: FSR amplifier 1

2. For the circuit in figure 3, if we want V_0 to go from 0 volts to 5 volts as the applied force goes from 0 kg to 1 kg, what value should R be?

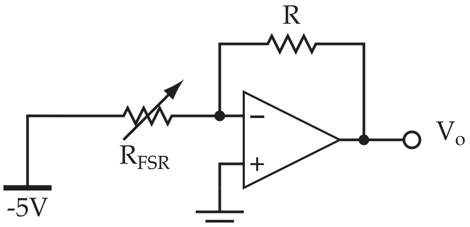

Figure 3: FSR amplifier 2

3. For the value of R you chose in (2), please plot, overlayed on your graph from (1), V_0 against force for the circuit shown in figure 3.
4. Under what circumstances would you use the first circuit rather than the second?

2 Instrumentation Amplifier

The circuit shown in figure 4 is a standard 3 op amp differential amplifier, also called an *instrumentation amplifier*. Its advantages over a single op amp differential amplifier are its much higher input impedance, balanced impedance between the two inputs, and a gain that can be changed by a single resistor. The questions below will walk you through an analysis of this useful amplifier.

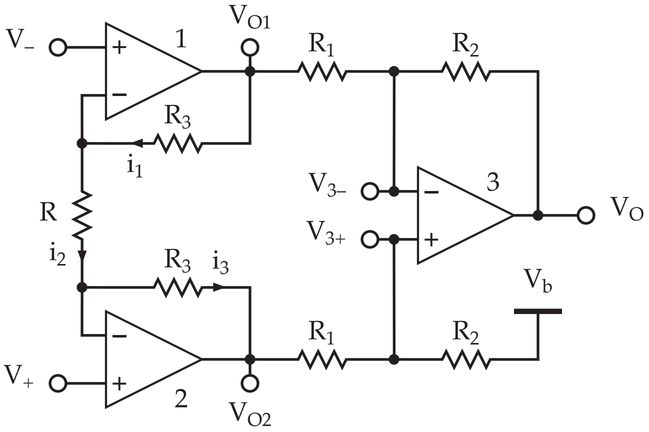

Figure 4: Instrumentation amplifier

Questions

1. What are the voltages at the negative terminals of op amps 1 and 2?
2. What is the current i_2 flowing through R in terms of V_+ and V_-?
3. What are the currents i_1 and i_3?
4. What is the voltage difference between V_{O1} and V_{O2} in terms of i_2, R, and R_3?
5. Substitute your result from (2) into (4) to find a relation for the difference between V_{O1} and V_{O2} in terms of R, R_3, V_+, and V_-.
6. What is V_{3-} in terms of V_{O1}, V_O, R_1, and R_2?
7. What is V_{3+} in terms of V_{O2}, V_b, R_1, and R_2?
8. Knowing that op amp 3 will hold V_{3+} equal to V_{3-}, find V_O as a function of V_{O1}, V_{O2}, V_b, R_1, and R_2.
9. Substitute your result from (5) into (8) to obtain the transfer function for this circuit.
10. Which single resistor would you vary to change the gain?
11. How would you bias this amplifier?

12. Consider the transfer function you found in (9). What useful function is the circuit performing? List two examples for how this may be used in a sensor-based system.

3 Strain Gauges

Strain gauges are inexpensive and accurate tools for measuring force and displacement. Since the level of displacement that they measure is so small, they are more often used as force sensors. By attaching the strain gauge to a flexible member of known geometry and material properties, the displacement to force characteristics can be derived. The simplest form of a flexible member is the cantilever beam, shown in figure 5.

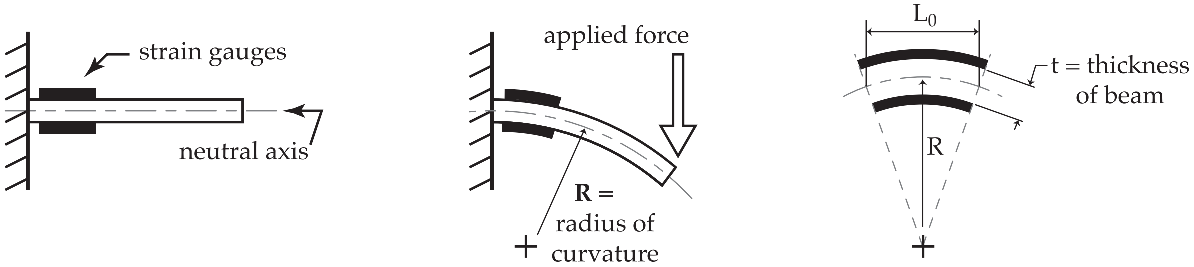

Figure 5: The geometry of strain gauges on a cantilevered beam

The applied force bends the beam, and creates a radius of curvature which decreases with the force, such that it is infinite (i.e. the beam is flat) when no force is applied.

The *neutral axis* is the line about which the beam bends. When a downward force is applied (as in figure 5), the material above the netural axis expands, while the material below the neutral axis is compressed. The neutral axis itself stays the same length.

Strain gauges are extremely thin, so we can neglect their thickness in relation to the beam thickness t. This simplification allows us to assume that the strain e induced in the strain gauge is exactly equal to the strain in the exterior surfaces of the beam. For some section with a neutral axis of length L_0, this strain is merely the change in length over the original length, as in equation 2. L_1 is the length at the surface of the beam.

$$e = \frac{L_1 - L_0}{L_0} \tag{2}$$

Questions

For this problem, assume we have strain gauges applied to both the top and bottom of a cantilever beam, as shown in figure 5, and that the strain gauge is thin compared to the beam thickness. The beam thickness $t = 1$ mm, and the minimum radius of curvature seen will be 20 cm. The strain gauges have an initial resistance $R_0 = 200\ \Omega$, and a gauge factor $S_e = 2$.

1. What is the peak strain e_{max} of the inner and outer strain gauges?

2. What is the resistance of the strain gauges under maximal load?
3. Using the strain gauges, design a temperature-compensated Wheatstone bridge. Assume both strain gauges are always at the same temperature.
4. What is the differential voltage, for a 5 volt excitation, across your Wheatstone bridge for both the unloaded and maximally loaded cases?
5. Design an amplifier circuit powered by a single 5 volt supply that will produce a reading of 0 volts for no applied force and 5 volts for the maximum force, with your Wheatstone bridge from (3).

4 Piezo Film Amplifier

For this problem, assume we are using the same cantilever beam setup shown in figure 5, but with PVDF piezo films rather than strain gauges. The geometry and maximum deflection of the beam will be the same. The piezo films are 30 μm thick, and 0.7 cm square. The 1-axis is along the length of the beam, and the 3-axis is perpendicular to the surface of the film.

Questions

Refer to the values in the MSI Piezo Technical Guide (URL provided earlier in this document) to answer the following questions.

1. What is the capacitance of each of the piezo films?
2. What is the maximum voltage produced by each of the piezo films?
3. Design a circuit with two resistors and a single op amp running from a single 5 volt supply. Your circuit should buffer the piezo signal, roll off frequencies below 100 Hz, have a gain of 1, and have its output centered around 2.5 volts.
4. Add two diodes and a resistor to your circuit from (3) to clamp the input voltage and protect the amplifier.